Self-attenuation

- Introduction
- Attenuation coefficients
- Self-attenuation
  - Simple analytical formula
  - Generalisation
  - Practical tools
- Examples

Introduction

- Emission of photons attenuated through the sample
- If sample different from the calibration (size, shape, density, chemical composition)
  - Reduction of the photon beam
  - « False » peak areas
  - Correction factor required to get the true activity
- Homogeneity ?

Calibration sources

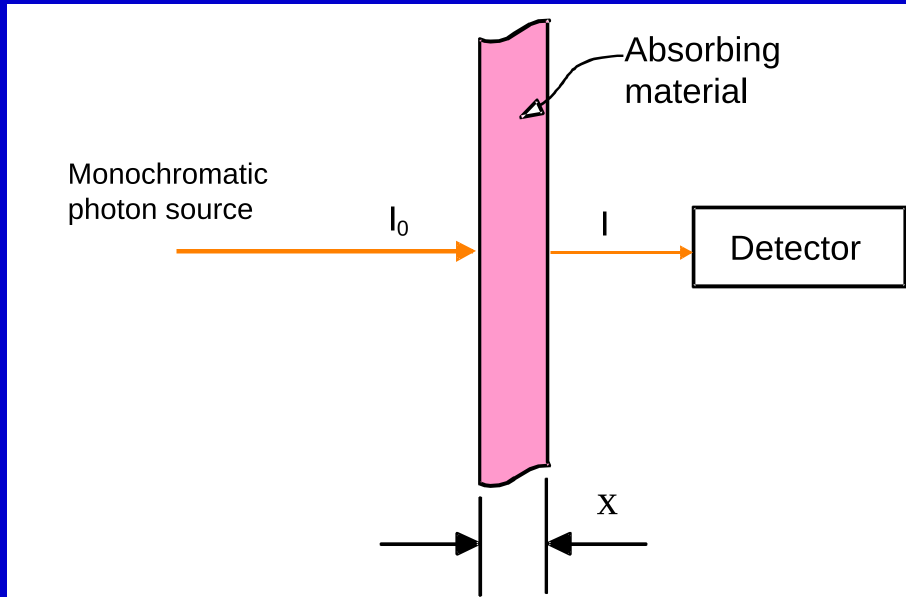
Attenuation coefficients

- Definition – Beer-Lambert law
- Tables
- Experimental measurement

Attenuation coefficients

Beer-Lamber law : attenuation of a narrow parallell photon beam

$$I(x) = I_0 \cdot e^{-\mu x} = I_0 \cdot e^{-\frac{\mu}{\rho}\rho x}$$



ρ = density (*g.cm*$^{-3}$)
μ = total linear attenuation coefficient of material i for enegy E (*cm*$^{-1}$)
ρ x = mass thickness (*g.cm*$^{-2}$)
μ / ρ = mass attenuation coefficient (*cm*^{2}*.g*$^{-1}$)

μ depends on E and Z

Practical parameter : attenuation coefficient

Partial interaction coefficients:

Photoelectric absorption: $\tau i(E)$ $\tau \approx const.Z^{4.5} \cdot E^{-3}$ (major at low energies)

Compton scattering: $\sigma i(E)$ $\sigma \approx const.\, Z \cdot E^{-1}$

Pair production effect: $\kappa i(E)$ $\kappa \approx const.\, Z^{2}$ (only if E > 1022 keV)

Interaction	Linear attenuation coefficient (cm^{-1})	Mass attenuation coefficient ($cm^{2} \cdot g^{-1}$)
Photoelectric	τ	τ/ρ
Compton	σ	σ/ρ
Pair production	κ	κ/ρ
Total	$\mu=\tau+\sigma+\kappa$	$\mu/\rho=\tau/\rho+\sigma/\rho+\kappa/\rho$

Depend on the energy, E, and the material (Z)
For practical use : tables function of Z and E

Tables : cross sections (*1 barn = 10^{-24} cm^{2}*) or mass attenuation (*$cm^{2}.g^{-1}$*)

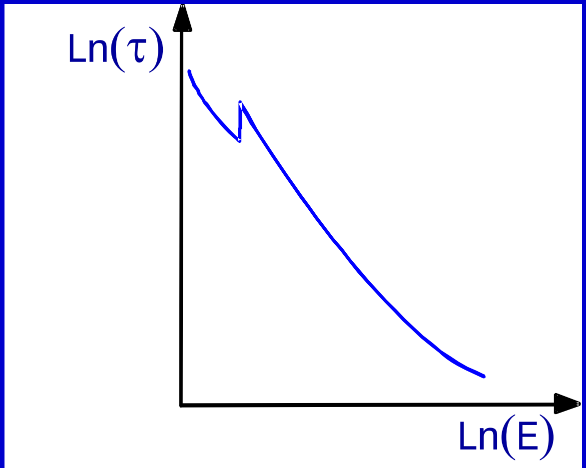
Photoelectric absorption coefficient = sum of photoelectric effect in each electronic shell (subshells):

$$\tau = \tau_K + (\tau_{L1} + \tau_{L2} + \tau_{L3}) + (\tau_{M1} + \tau_{M2} + \tau_{M3} + \tau_{M4} + \tau_{M5}) + \ldots$$

If E < binding energy of shell i, $\tau_i = 0$

For E = Ei : absorption discontinuity: maximum ionisation probability in shell i

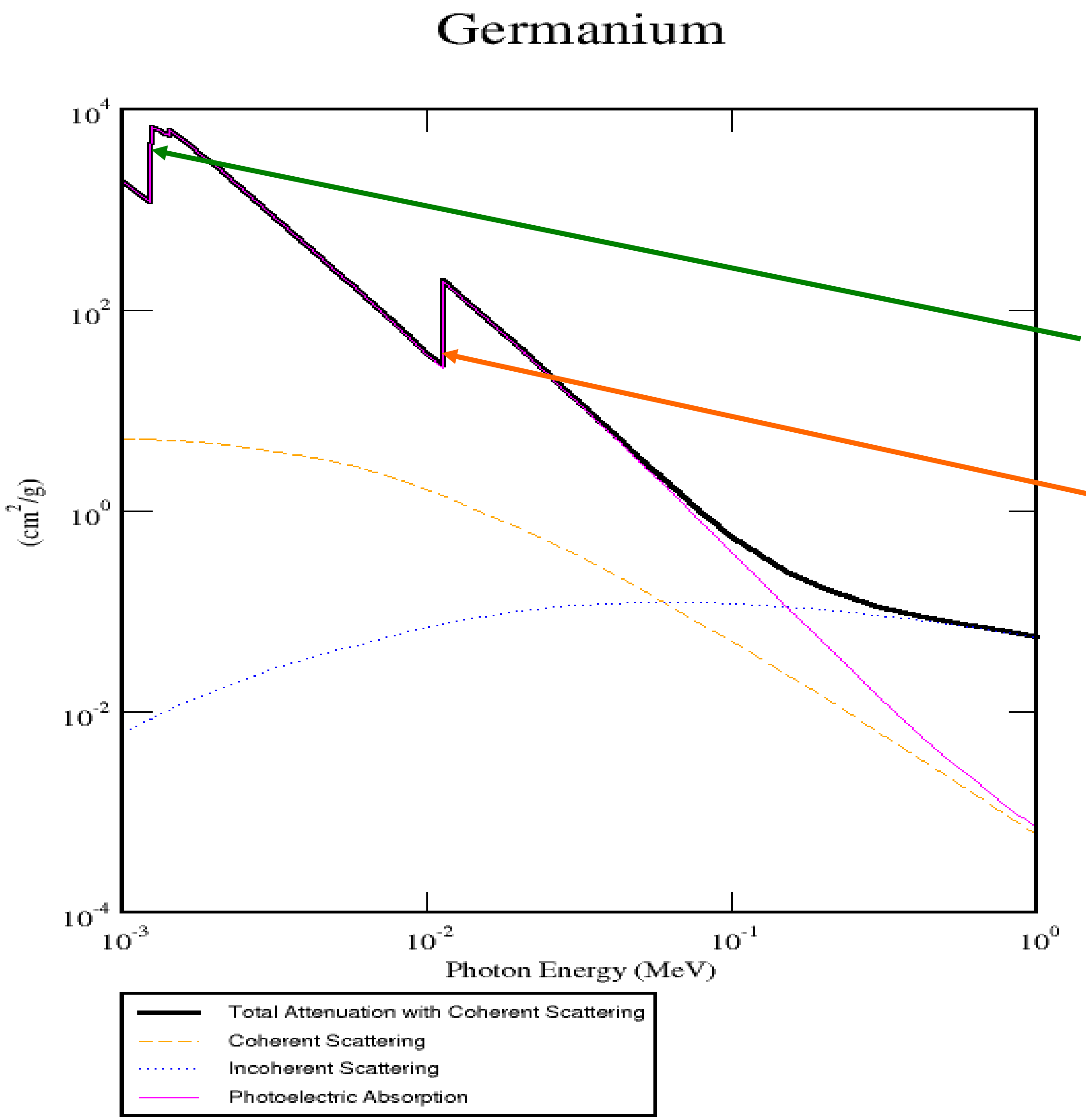
τ variation versus the energy shows discontinuities corresponding to binding energies of electrons shells and subshells K, L, M...



Since $\mu = \tau + \sigma + \kappa$

μ has the same discontinuities, function of the material atomic structure (Z)

Germanium mass attenuation coefficient

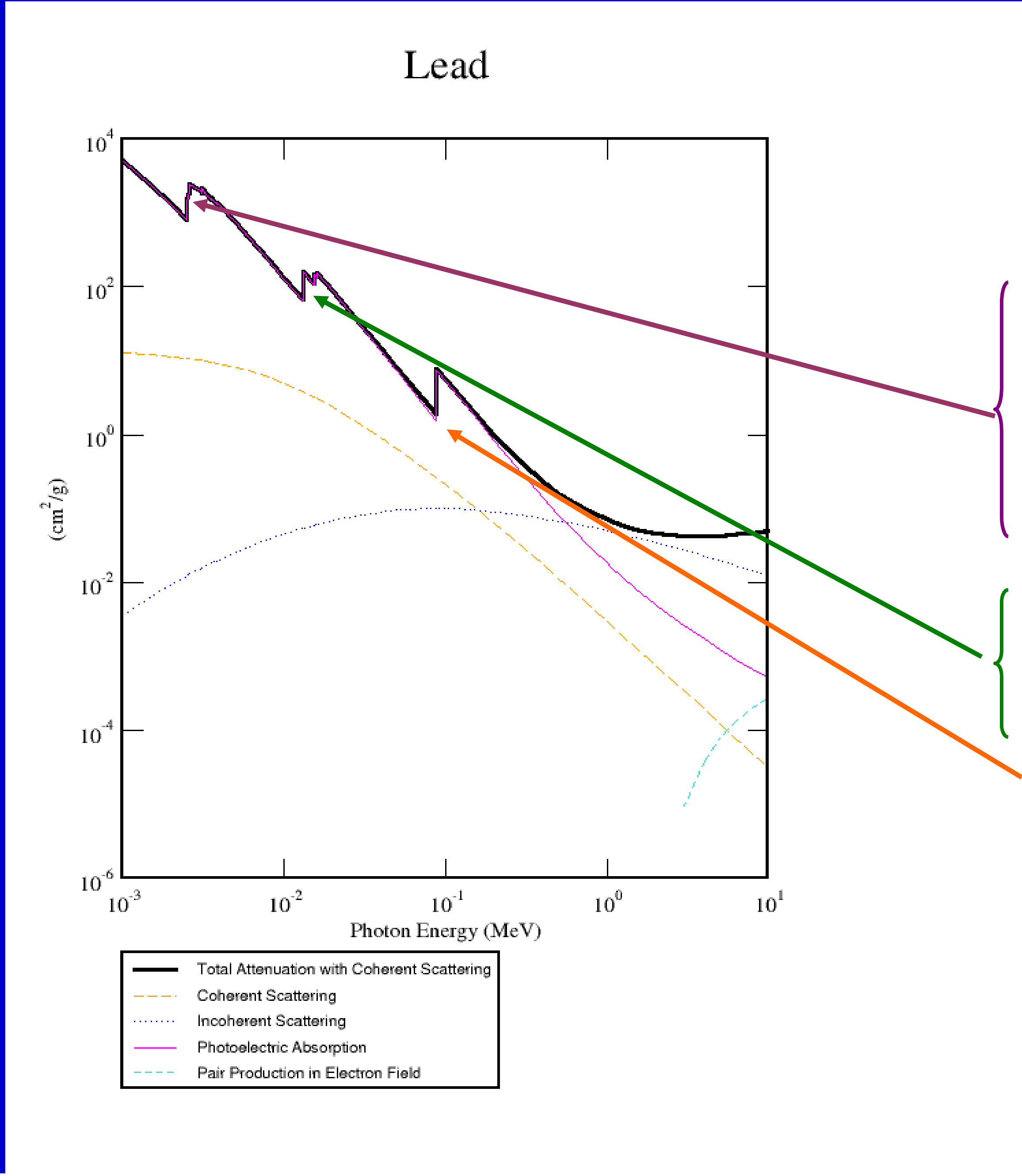


Ge binding energies :

L1: 1.4143
L2: 1.2478
L3: 1.2167

K: 11.1031

Lead mass attenuation coefficient



Pb binding energies :

M1: 3.8507
M2: 3.5542
M3: 3.0664
M4: 2.5856
M5: 2.4840

L1: 15.8608
L2: 15.2000
L3: 13.0352

K: 88.0045

Mass attenuation coefficients

- Composition known -> calculation
- Composition unknown -> measurement

- Calculation: Attenuation coefficient table
  – XCOM (NIST Database)
  – Example for HCl 1N

XCOM : mixture

- Defining the mass fraction of each compound for HCl 1N:
- Matrix : HCl 1N = 1 mole of HCl in 1 liter of solution
- HCl 1N density = 1.016 (1L = 1016 g)
- Mass of one HCl mole = 1+ 35.45 = 36.45 g
- Resulting input parameters for XCOM
- Compound 1: HCl
- Mass fraction: 36.45
- Compound 2: H_2O
- Mass fraction = 1016 – 36.45 = 979.55

XCOM

NIST
National Institute of Standards and Technology
Physics Laboratory

XCOM

Element/Compound/Mixture Selection

In this database, it is possible to obtain photon cross section data for a single element, compound, or mixture (a combination of elements and compounds). Please fill out the following information:

Help

Identify material by:

- ○ Element
- ○ Compound
- ◉ Mixture

Method of entering additional energies: (optional)

- ◉ Enter additional energies by hand
- ○ Additional energies from file (*Note: Your browser must be file-upload compatible*)

[Submit Information] [Reset]

XCOM

http://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html

XCOM input-output

Fill out the form to select the data to be displayed:

Help

Enter the formulae and relative weights separated by a space for each compound. One compound per line. For example:

```
H2O 0.9
NaCl 0.1
```

Note: Weights not summing to 1 will be normalized.

```
HCl 36.45
H2O 979.55
```

Optional output title: Mixture HCl 1N

Graph options:

- [ ] Total Attenuation with Coherent Scattering
- [x] Total Attenuation without Coherent Scattering
- [ ] Coherent Scattering
- [x] Incoherent Scattering
- [x] Photoelectric Absorption
- [ ] Pair Production in Nuclear Field
- [ ] Pair Production in Electron Field
- [ ] None

Additional energies in MeV: (optional) (up to 75 allowed)

Note: Energies must be between 0.001 - 100000 MeV (1 keV - 100 GeV) (only 4 significant figures will be used). One energy per line. Blank lines will be ignored.

```
0.122
0.320
```

- [x] **Include the standard grid**

Energy Range:

Minimum: 0.01 MeV

Maximum: 10 MeV

Submit Information | Reset

Constituents (Atomic Number : Fraction by Weight)

```
Z=1    : 0.108875
Z=8    : 0.856240
Z=17   : 0.034884
```

To download data in spreadsheet (array) form, choose a delimiter and use the checkboxes in the table heading. After down

Delimiter:

- (•) space
- ( ) | (vertical bar)
- ( ) tab
- ( ) newline

Download data | Reset

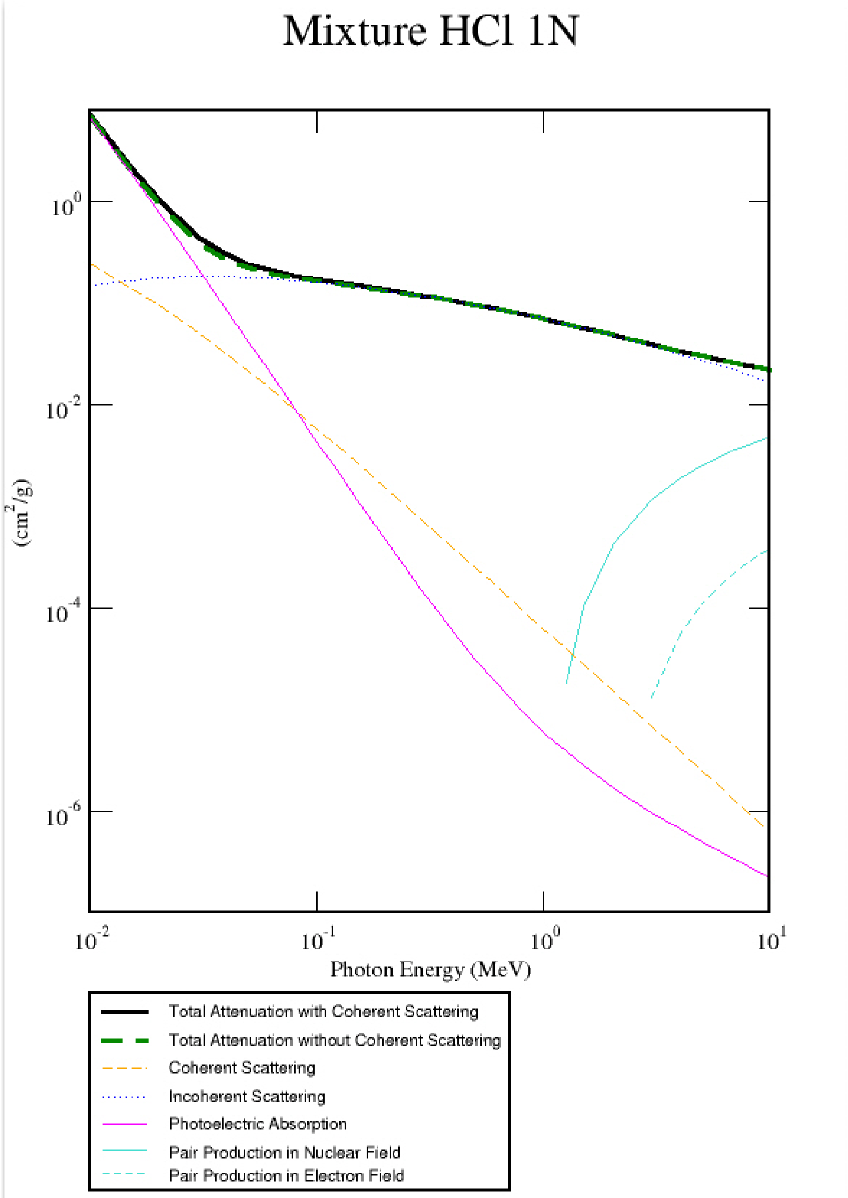
Edge	(required) Photon Energy	Scattering		Photoelectric Absorption	Pair Production		Total Attenuation	
		Coherent	Incoherent		In Nuclear Field	In Electron Field	With Coherent Scattering	Without Coherent Scattering
	MeV	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g
	1.000E-02	2.48E-01	1.53E-01	6.73E+00	0.00E+00	0.00E+00	7.14E+00	6.89E+00
	1.500E-02	1.44E-01	1.68E-01	1.92E+00	0.00E+00	0.00E+00	2.24E+00	2.09E+00
	2.000E-02	9.57E-02	1.76E-01	7.80E-01	0.00E+00	0.00E+00	1.05E+00	9.55E-01
	3.000E-02	5.08E-02	1.82E-01	2.15E-01	0.00E+00	0.00E+00	4.47E-01	3.96E-01
	4.000E-02	3.12E-02	1.82E-01	8.52E-02	0.00E+00	0.00E+00	2.98E-01	2.67E-01
	5.000E-02	2.11E-02	1.79E-01	4.14E-02	0.00E+00	0.00E+00	2.42E-01	2.21E-01
	6.000E-02	1.52E-02	1.76E-01	2.29E-02	0.00E+00	0.00E+00	2.14E-01	1.99E-01
	8.000E-02	8.93E-03	1.69E-01	8.99E-03	0.00E+00	0.00E+00	1.87E-01	1.78E-01
	1.000E-01	5.86E-03	1.62E-01	4.35E-03	0.00E+00	0.00E+00	1.72E-01	1.66E-01
	1.220E-01	4.00E-03	1.55E-01	2.28E-03	0.00E+00	0.00E+00	1.61E-01	1.57E-01
	1.500E-01	2.68E-03	1.47E-01	1.17E-03	0.00E+00	0.00E+00	1.51E-01	1.48E-01
	2.000E-01	1.53E-03	1.35E-01	4.67E-04	0.00E+00	0.00E+00	1.37E-01	1.35E-01

Terminé

XCOM Results

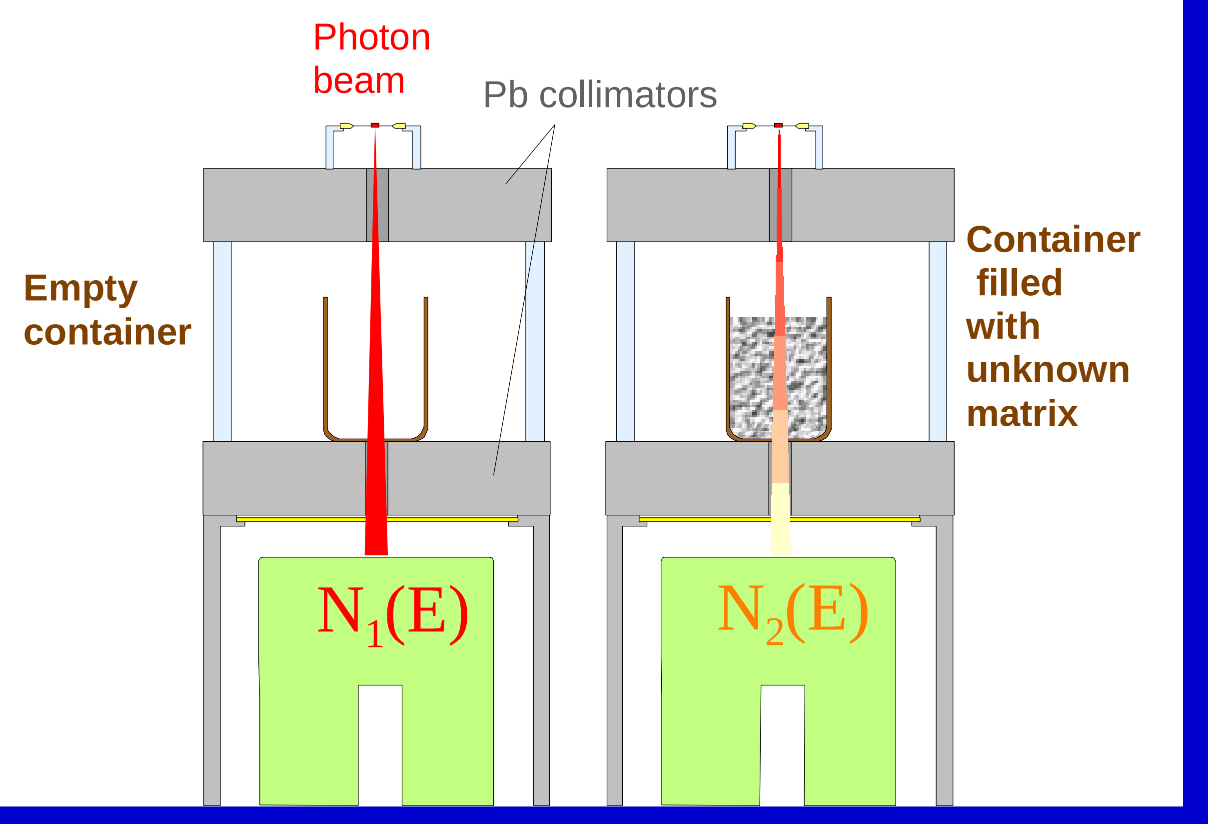
tab
newline
Download data Reset

Edge	(required) Photon Energy	Scattering		Photoelectric Absorption	Pair Production		Total Attenuation	
		Coherent	Incoherent		In Nuclear Field	In Electron Field	With Coherent Scattering	Without Coherent Scattering
	MeV	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g	cm^2/g
	1.000E-03	1.43E+00	1.31E-02	4.03E+03	0.00E+00	0.00E+00	4.03E+03	4.03E+03
	1.500E-03	1.32E+00	2.65E-02	1.36E+03	0.00E+00	0.00E+00	1.36E+03	1.36E+03
	2.000E-03	1.20E+00	4.14E-02	6.10E+02	0.00E+00	0.00E+00	6.11E+02	6.10E+02
	2.822E-03	9.89E-01	6.52E-02	2.27E+02	0.00E+00	0.00E+00	2.28E+02	2.27E+02
17 K	2.822E-03	9.89E-01	6.52E-02	2.78E+02	0.00E+00	0.00E+00	2.79E+02	2.78E+02
	3.000E-03	9.47E-01	6.99E-02	2.36E+02	0.00E+00	0.00E+00	2.37E+02	2.36E+02
	4.000E-03	7.41E-01	9.32E-02	1.04E+02	0.00E+00	0.00E+00	1.04E+02	1.04E+02
	5.000E-03	5.86E-01	1.11E-01	5.40E+01	0.00E+00	0.00E+00	5.47E+01	5.41E+01
	6.000E-03	4.75E-01	1.24E-01	3.15E+01	0.00E+00	0.00E+00	3.21E+01	3.16E+01
	8.000E-03	3.31E-01	1.42E-01	1.33E+01	0.00E+00	0.00E+00	1.38E+01	1.34E+01
	1.000E-02	2.48E-01	1.53E-01	6.73E+00	0.00E+00	0.00E+00	7.14E+00	6.89E+00
	1.500E-02	1.44E-01	1.68E-01	1.92E+00	0.00E+00	0.00E+00	2.24E+00	2.09E+00
	2.000E-02	9.57E-02	1.76E-01	7.80E-01	0.00E+00	0.00E+00	1.05E+00	9.55E-01
	3.000E-02	5.08E-02	1.82E-01	2.15E-01	0.00E+00	0.00E+00	4.47E-01	3.96E-01
	4.000E-02	3.12E-02	1.82E-01	8.52E-02	0.00E+00	0.00E+00	2.98E-01	2.67E-01
	5.000E-02	2.11E-02	1.79E-01	4.14E-02	0.00E+00	0.00E+00	2.42E-01	2.21E-01
	6.000E-02	1.52E-02	1.76E-01	2.29E-02	0.00E+00	0.00E+00	2.14E-01	1.99E-01
	8.000E-02	8.93E-03	1.69E-01	8.99E-03	0.00E+00	0.00E+00	1.87E-01	1.78E-01
	1.000E-01	5.86E-03	1.62E-01	4.35E-03	0.00E+00	0.00E+00	1.72E-01	1.66E-01
	1.500E-01	2.68E-03	1.47E-01	1.17E-03	0.00E+00	0.00E+00	1.51E-01	1.48E-01
	2.000E-01	1.53E-03	1.35E-01	4.67E-04	0.00E+00	0.00E+00	1.37E-01	1.35E-01
	3.000E-01	6.84E-04	1.17E-01	1.34E-04	0.00E+00	0.00E+00	1.18E-01	1.18E-01
	4.000E-01	3.86E-04	1.05E-01	5.76E-05	0.00E+00	0.00E+00	1.06E-01	1.05E-01
	5.000E-01	2.48E-04	9.62E-02	3.12E-05	0.00E+00	0.00E+00	9.65E-02	9.63E-02
	5.140E-01	2.34E-04	9.51E-02	2.90E-05	0.00E+00	0.00E+00	9.54E-02	9.52E-02
	6.000E-01	1.72E-04	8.90E-02	1.95E-05	0.00E+00	0.00E+00	8.92E-02	8.91E-02
	8.000E-01	9.69E-05	7.82E-02	9.83E-06	0.00E+00	0.00E+00	7.84E-02	7.83E-02
	1.000E+00	6.20E-05	7.04E-02	6.11E-06	0.00E+00	0.00E+00	7.04E-02	7.04E-02
	1.022E+00	5.94E-05	6.96E-02	5.73E-06	0.00E+00	0.00E+00	6.97E-02	6.96E-02
	1.173E+00	4.51E-05	6.50E-02	4.30E-06	6.51E-06	0.00E+00	6.51E-02	6.50E-02
	1.250E+00	3.97E-05	6.29E-02	3.89E-06	1.86E-05	0.00E+00	6.30E-02	6.29E-02



Experimental measurement

Principle : Use the unknown matrix and a collimated photon beam



Two successive measurements

- Empty container
- Container filled with unknown matrix with thickness x

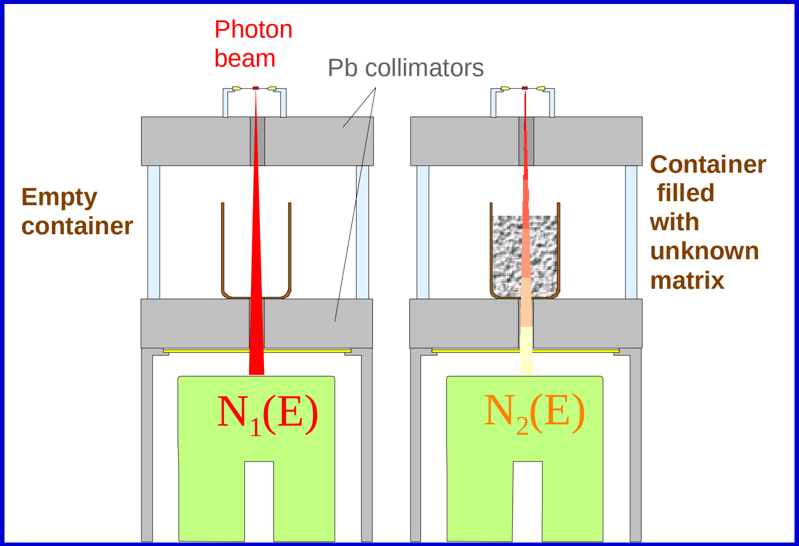
Experimental measurement

For each energy:

$$N_2(E) = N_1(E) \, . \, exp \, (-\mu(E) \, . \, x)$$

Thus:

$$\mu(E) = \frac{1}{x} \ln\left(\frac{N_2(E)}{N_1(E)}\right)$$

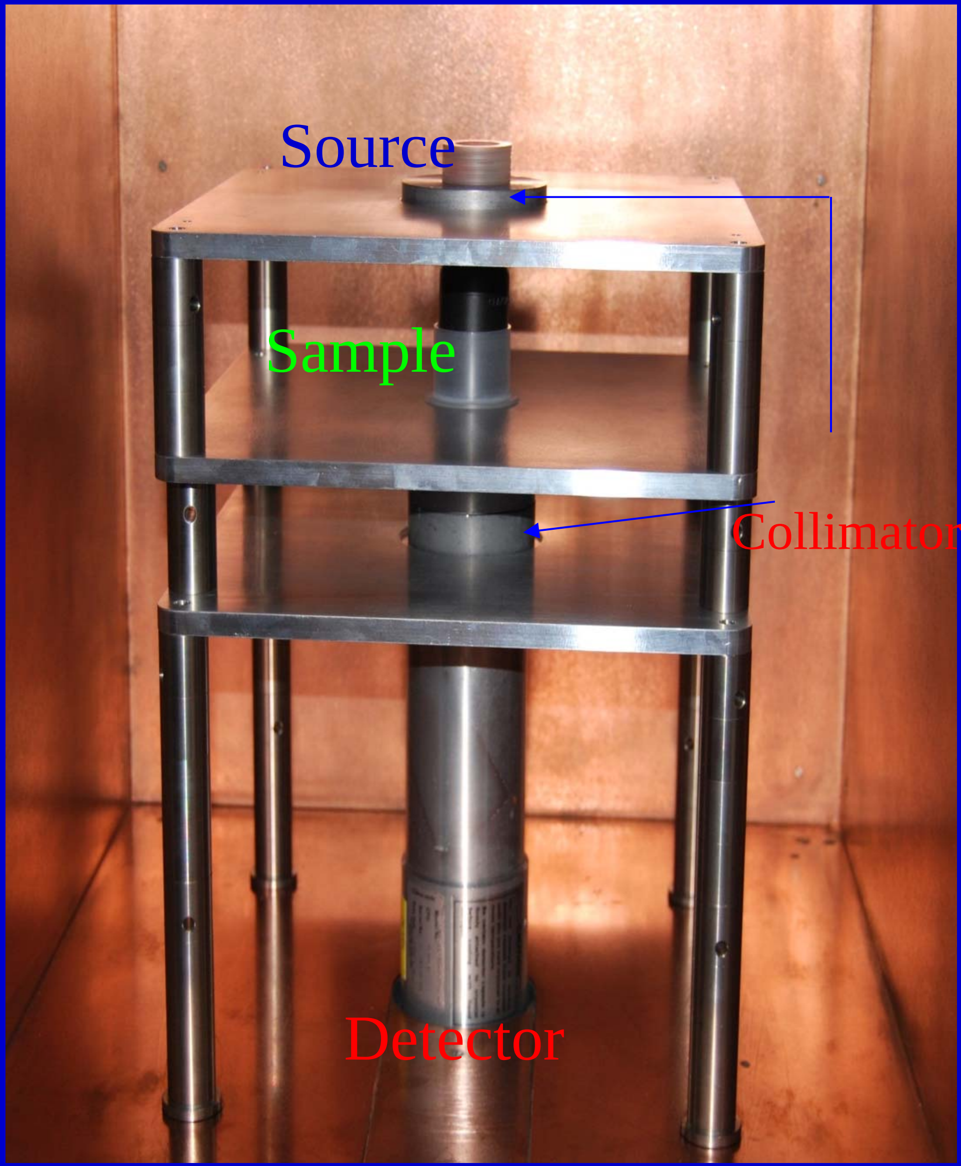


Associated relative uncertainty:

$$\frac{u^2(\mu)}{\mu^2} = \frac{u^2(x)}{x^2} + \frac{1}{\ln^2\left(\frac{N_0(E)}{N(E)}\right)} \cdot \left(\frac{u^2(N_0(E))}{N_0^2(E)} + \frac{u^2(N(E))}{N^2(E)}\right)$$

Experimental arrangement

New experimental arrangement



Experimental measurement

Problems:

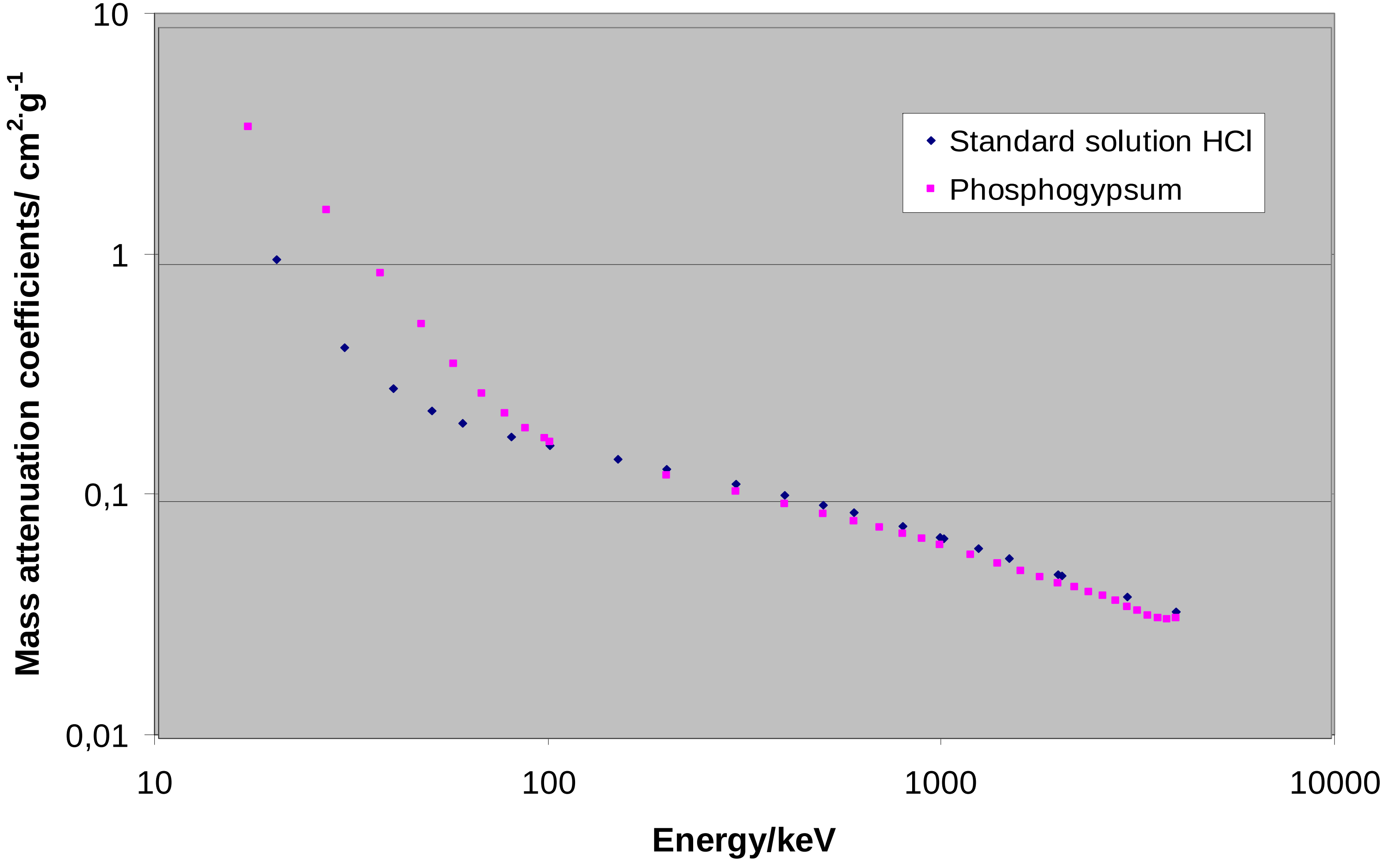
- Single line gamma emitters should be used to avoid coincidence summing effects
- At low energy – small angle Compton scattering contribution
=> the collimated source and the sample far from detector
=> High intensity sources required – storage problem ?

- Time consuming

Measured linear attenuation coefficient

x = 2.5 cm

Energy/keV	FEP counting rate (spectrum 1) = N_1/t_1	FEP counting rate (spectrum 2) = N_2/t_2	μ_m (cm^{-1}) = $(1/x)\ \ln((N_1/t_1)/(N_2/t_2))$
39	39.49	4.81	0.842
45	10.26	2.18	0.619
53	5.18	1.612	0.468
59	78.70	32.29	0.356
81	89.17	50.18	0.230
122	17.20	11.45	0.163
244	2.44	1.78	0.128
276	6.19	4.59	0.120
302	14.70	11.15	0.110
344	6.67	5.08	0.109
356	48.52	37.25	0.106
383	7.25	5.63	0.101
662	60.06	48.78	0.083



10
1
0,1
0,01
Mass attenuation coefficients/ cm2·g-1
Standard solution HCl
Phosphogypsum
10
100
1000
10000
Energy/keV

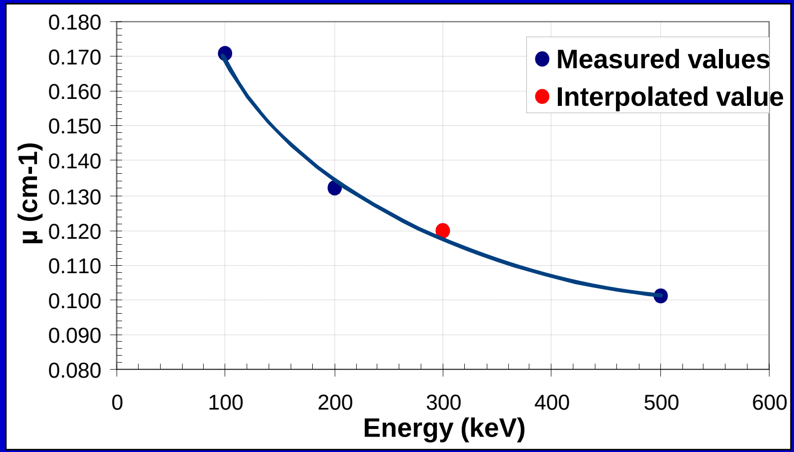
Interpolated attenuation coefficient

Example: attenuation in a 10 cm thick matrix for 300 keV energy

Mesurement of µ for some energies:

Energy/*keV*	100	200	500
N_1 (s^{-1})	1000	600	200
N_2 (s^{-1})	180	160	73
N_1 / N_2	0.180	0.267	0.365
µ (*cm*$^{-1}$)	0.171	0.132	0,101

Interpolation for E=300 keV:



Linear interpolation: µ = 0.122 cm^{-1}

Logarithmic interpolation: µ = 0.117 cm^{-1}

Self attenuation

- Simple formula
- Generalisation

Self-attenuation in a volume sample

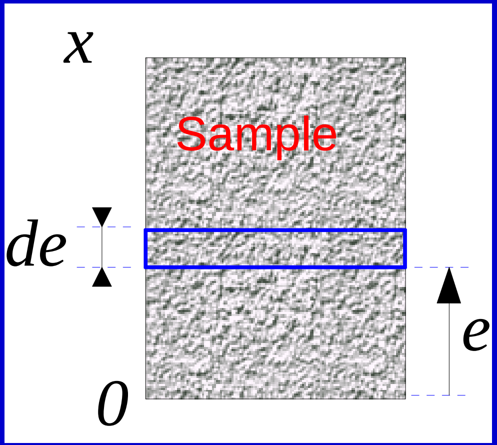
Intrinsic photon flux: I_0 (What you wish to know to derive the activity)
Emitted photon flux: I (What is recorded by the detector)

For a thin layer, with thickness de:

$$dI_0 = \frac{I_0}{x} de$$

Only true if the sample is homogeneous !

This partial photon flux is attenuated through thickness e:



$$dI = \frac{I_0}{x} \exp(-\mu \cdot e) de$$

μ = linear attenuation coefficient (cm^{-1})

For the whole volume with thickness *x* :

$$I = \int_0^x I_0 \exp(-\mu \cdot e) \cdot \frac{de}{x} = I_0 \cdot \frac{1 - \exp(-\mu \cdot x)}{\mu \cdot x}$$

Self-attenuation in a volume sample

$$I = I_0 \cdot \frac{1-\exp(-\mu \cdot x)}{\mu \cdot x}$$

Intrinsic photon flux: I_0 (What you wish to know to derive the activity)
Emitted photon flux: I (What is recorded by the detector)

Self-attenuation: $$C_{att} = \frac{1-\exp(-\mu x)}{\mu x}$$

Cutshall et al. NIM 206 (1983) 309

- Approximation for a thin source (µx <1) :

$$I = I_0 \cdot \left(1-\frac{\mu \cdot x}{2}\right)$$

$$C_{att} = 1-\frac{\mu \cdot x}{2}$$

Interpolated attenuation coefficient

Example: attenuation in a 10 cm thick matrix for 300 keV energy

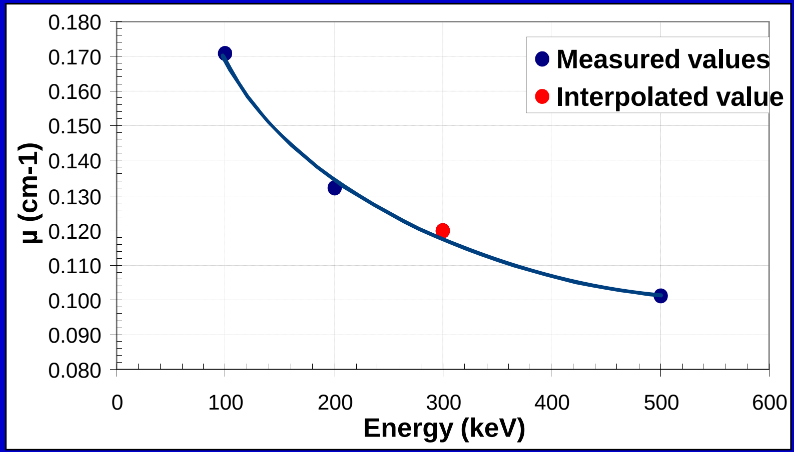
- Mesurement of µ for some energies:

Energy/*keV*	100	200	500
N_1 (s^{-1})	1000	600	200
N_2 (s^{-1})	180	160	73
N_1 / N_2	0.180	0.267	0.365
µ (*cm^{-1}*)	0.171	0.132	0,101

- Interpolation for E=300 keV:

Linear interpolation: µ = 0.122 cm^{-1}

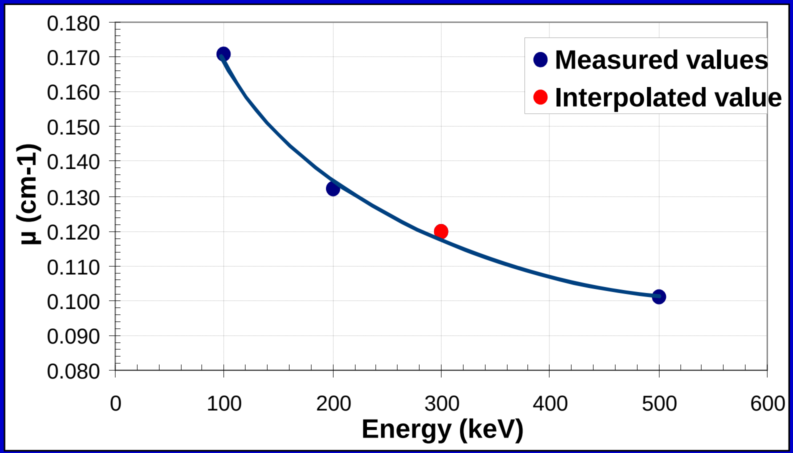
Logarithmic interpolation: µ = 0.117 cm^{-1}



Interpolated attenuation coefficient

Example: self-attenuation in a 10 cm thick matrix for 300 keV energy

- Interpolation for E=300 keV :



Linear interpolation: $\mu = 0.122\ \text{cm}^{-1}$

Logarithmic interpolation: $\mu = 0.117\ \text{cm}^{-1}$

- Self-attenuation at 300 keV :

$$C_{att} = \frac{1 - \exp(-\mu_{300} \cdot x)}{\mu_{300} \cdot x} = \frac{1 - \exp(-0.117 \cdot 10)}{0.117 \cdot 10} = 0.589$$

Self-attenuation in a volume sample

Self-attenuation: $C_{att} = \frac{1-\exp(-\mu x)}{\mu x}$

- If the measured sample is subject to attenuation and the calibration source is not, a correction factor must be applied to the peak area that is :1/C_{Satt}

$$C_{self} = C_{att}{}^{-1} = \frac{\mu x}{1-\exp(-\mu x)}$$

- If both are subject to self-attenuation, the corrective factor is the ratio of the self attenuation for each material

$$C_{self} = \frac{[C_{att}]^{-1}_{mes}}{[C_{att}]^{-1}_{cal}} = \frac{\left[\frac{\mu x}{1-\exp(-\mu x)}\right]_{mes}}{\left[\frac{\mu x}{1-\exp(-\mu x)}\right]_{cal}}$$

Mes = measured sample

Cal : calibration source

Self-attenuation in a volume sample

- Can also be computed as a transfer factor from an efficiency calibration established reference material to measure a different material (in the same geometry)

$$\varepsilon_{mes} = \varepsilon_{cal} \cdot f_{Self}$$

- Thus the efficiency transfer factor is:

$$f_{Self} = \frac{[C_{att}]_{mes}}{[C_{att}]_{cal}} = \frac{\left[\frac{1-\exp(-\mu x)}{\mu x}\right]_{mes}}{\left[\frac{1-\exp(-\mu x)}{\mu x}\right]_{cal}}$$

Mes = measured sample

Cal : calibration source

Transfer from an efficiency calibration established with a liquid source (filled with 10 cm HCl) for matrixes silica and sand:

$$\varepsilon_{mes} = \varepsilon_{cal} \cdot f_{Self} = \varepsilon_{cal} \cdot \frac{[C_{att}]_{mes}}{[C_{att}]_{cal}}$$

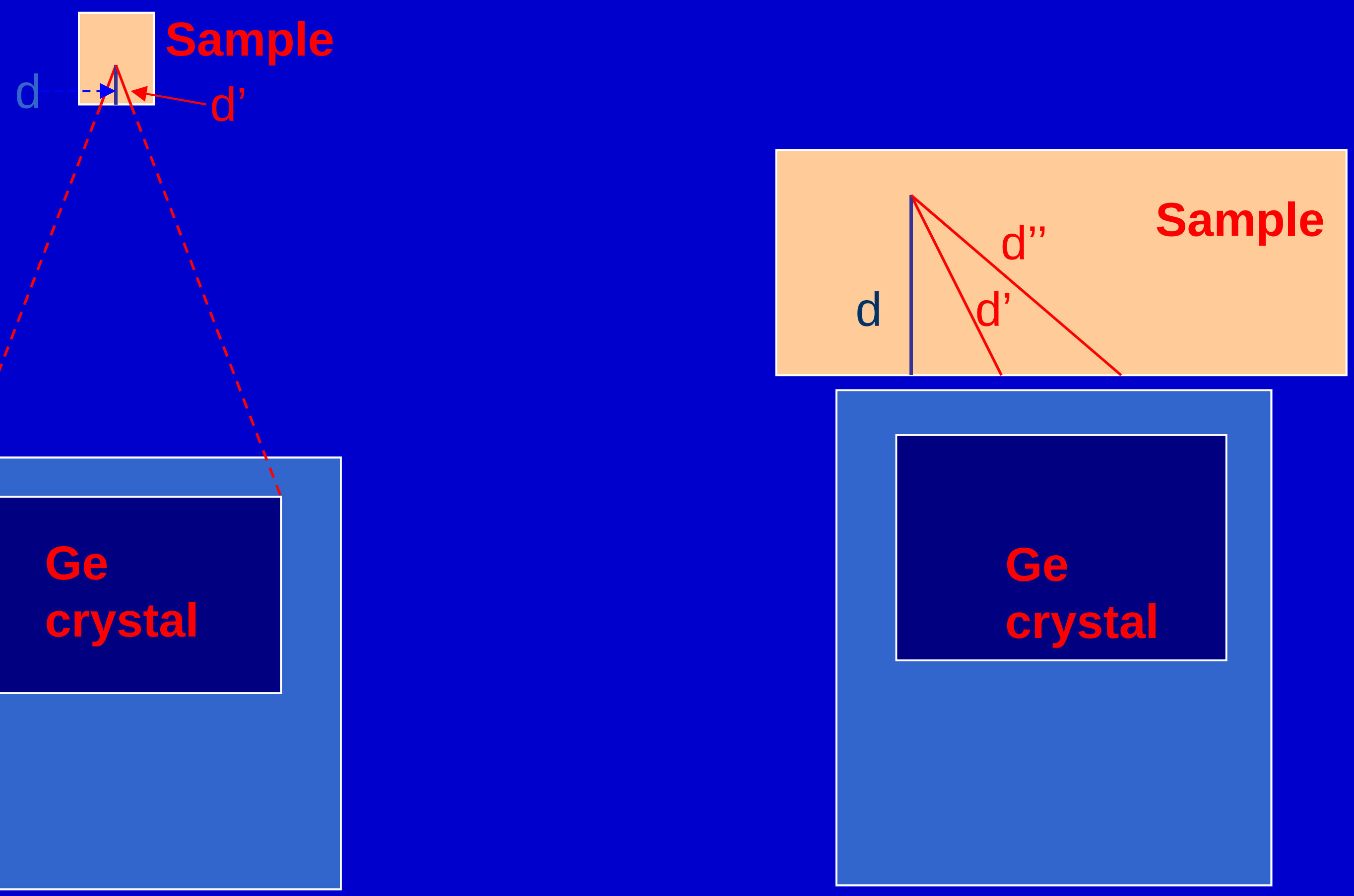
Densities:

water/HCl = 1.016
silica = 0.25
sand/resin = 1.54

Energy (*keV*)	100	200	300	500
μ_{HCl} $(cm^2.g^{-1})$	0.171	0.137	0.119	0.0969
μ_{HCl} (cm^{-1})	0.174	0.139	0.121	0.0985
$[C_{att}]cal$	0.474	0.540	0.580	0.636
μ_{silica} $(cm^2.g^{-1})$	0.168	0.125	0.108	0.0874
μ_{silica} (cm^{-1})	0.042	0.031	0.027	0.022
$[C_{att}]$	0.817	0.860	0.876	0.898
f_{Self} (silica)	1.72	1.59	1.51	1.41
μ_{sand} $(cm^2.g^{-1})$	0.170	0.131	0.113	0.0919
μ_{sand} (cm^{-1})	0.262	0.202	0.174	0.142
$[C_{att}]$	0.354	0.429	0.474	0.534
f_{Self} (sand)	0.75	0.79	0.82	0.84

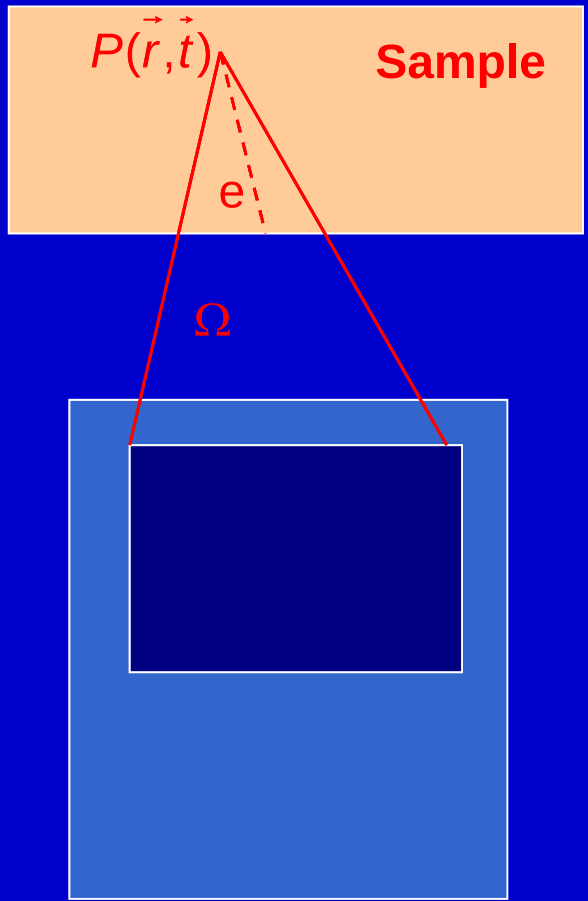
General formula

- Realistic if the source is far from the detector (parallel beam - normal incidence) - small source
- Not true for environment measurements (d', d" » d)



General formula

- Must consider all possible trajectories for each point of the volume sample -> integration over solid angle and sample volume



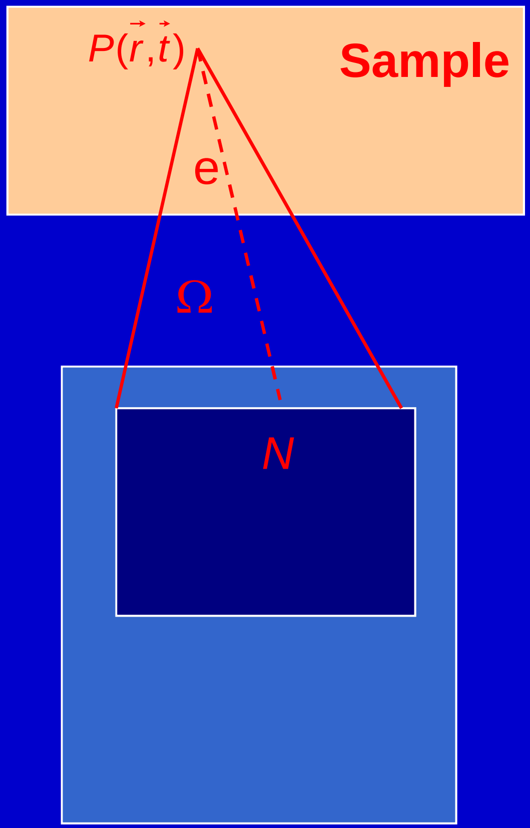
$$C_{att} = \frac{\int_V dV \int_\Omega \exp(-\mu(E) \cdot e(\vec{r},\vec{t}))\, d\Omega}{\int_V dV \int_\Omega d\Omega}$$

Point P with position r, and emission direction t

e: path in the sample matrix

Add the container absorption and probability of full-absorption of the photon in the detector active volume

General formula



$$C_{att} = \frac{\int_V dV \int_\Omega \exp(-\mu(E)\cdot e(\vec{r},\vec{t}))\cdot T(E,\vec{r},\vec{t})\cdot P(E,\vec{r},\vec{t})\cdot d\Omega}{\int_V dV \int_\Omega T(E,\vec{r},\vec{t})\cdot P(E,\vec{r},\vec{t})\cdot d\Omega}$$

Denominator = « self attenuation » for a transparent sample

μ(E) : attenuation coefficient of the sample material for the energy E

e : trajectory through the sample

T : Transmission through absorbers (container, detector window, …)

P : Probability of full-energy absorption in the detector

This correction can be numerically computed (Gauss-Legendre integration)

Efficiency transfer factor

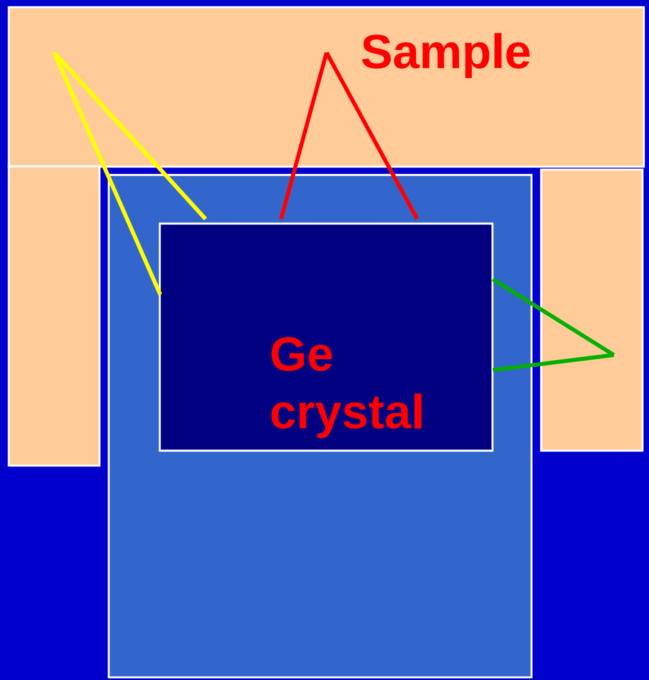
- Transfer factor from an efficiency calibration established reference material to measure a different material (in the same geometry)

$$f_{Self} = \frac{[C_{att}]_{mes}}{[C_{att}]_{cal}}$$

$$\varepsilon_{mes} = \varepsilon_{cal} \cdot f_{Self} = \varepsilon_{cal} \cdot \frac{[C_{att}]_{mes}}{[C_{att}]_{cal}}$$

This transfer factor can be numerically computed (Gauss-Legendre integration)

Self-attenuation in Marinelli geometry



$$C_{att} = \frac{\int_V dV \int_\Omega \exp(-\mu(E) \cdot e(\vec{r},\vec{t})) \cdot T(E,\vec{r},\vec{t}) \cdot P(E,\vec{r},\vec{t}) \cdot d\Omega}{\int_V dV \int_\Omega T(E,\vec{r},\vec{t}) \cdot P(E,\vec{r},\vec{t}) \cdot d\Omega}$$

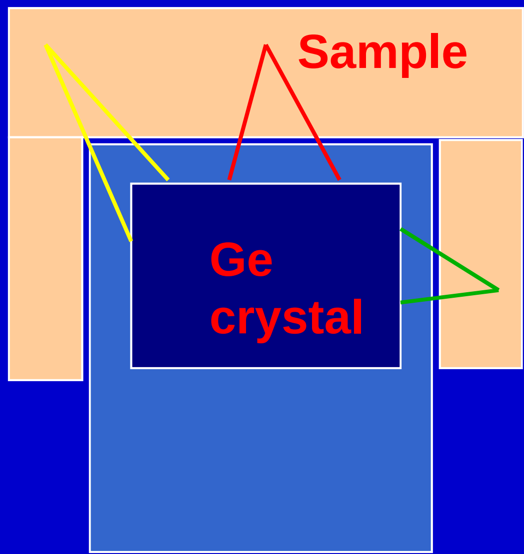
The general expression must be extended to different parts of the sample, according to the path of the photons

Numerical integration using different volumes

This correction can be numerically computed (Gauss-Legendre integration)

Monte Carlo simulation

- Self attenuation can be computed using Monte Carlo methods
  - General codes (GEANT, MCNP, PENELOPE, etc)
  - Dedicated software (DETEFF, GESPECOR, etc.)



- Any geometry (including non-cylindrical symmetry) can be considered

- Time-consuming ? Dedicated sofware are optimized

Practical tools

Methods for self-attenuation correction

- Empirical methods – simplified computing
- Analytic approach
  - ANGLE
  - ETNA , etc.
- Monte Carlo methods
  - DETEFF
  - GESPECOR
  - General codes (GEANT, PENELOPE, MCNP)

Examples

- Importance of the material density
- Influence of the filling height
- Change of matrix

Self –attenuation in silica

- Silica low density (0.25 g.cm^{-3})
- Sand (mainly silica) (2.5 g.cm^{-3})
- Thickness 1 cm

Radionuclide	Energy/keV	Mass att coefficient ($cm^2.g^{-1}$)	Self-attenuation Silica	Self-attenuation Sand
^{210}Pb	46.5	0.356	0.957	0.662
^{137}Cs	661.7	0.0773	0.990	0.909
^{40}K	1460.8	0.0526	0.993	0.937

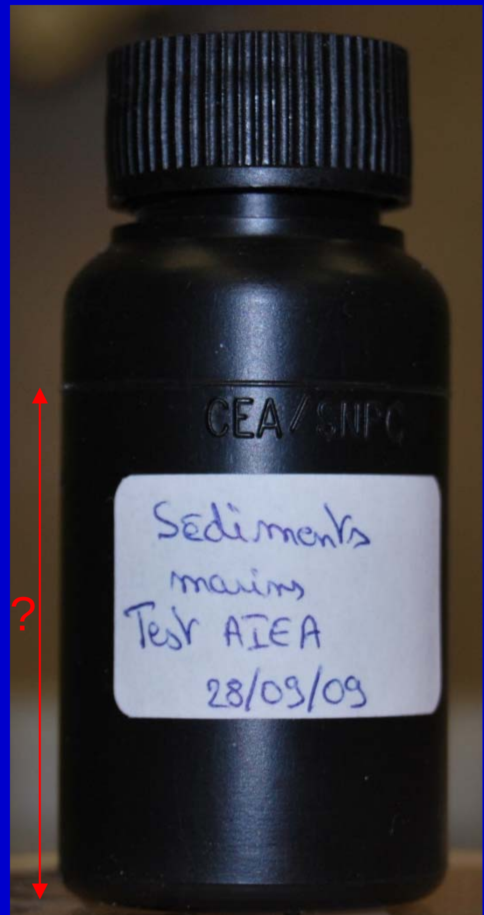
Self –attenuation in steel

- Fe (7.5 $g.cm^{-3}$)

Radionuclide	Energy/keV	Mass att coefficient ($cm^2.g^{-1}$)	Self-attenuation 1 cm	Self-attenuation 1 mm	Self-attenuation 0.1 mm
^{210}Pb	46.5	2.39	0.056	0.465	0.915
^{137}Cs	661.7	0.0735	0.769	0.973	0.997
^{40}K	1460.8	0.0495	0.835	0.982	0.998

For the low energies, only the very first thickness contributes

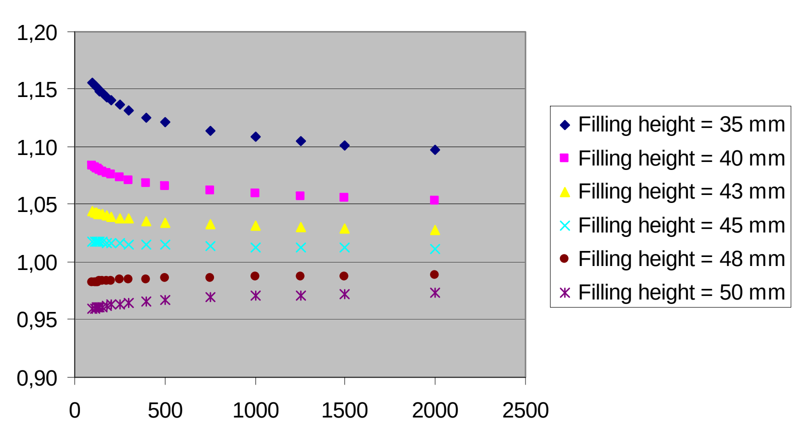
Influence of the filling height



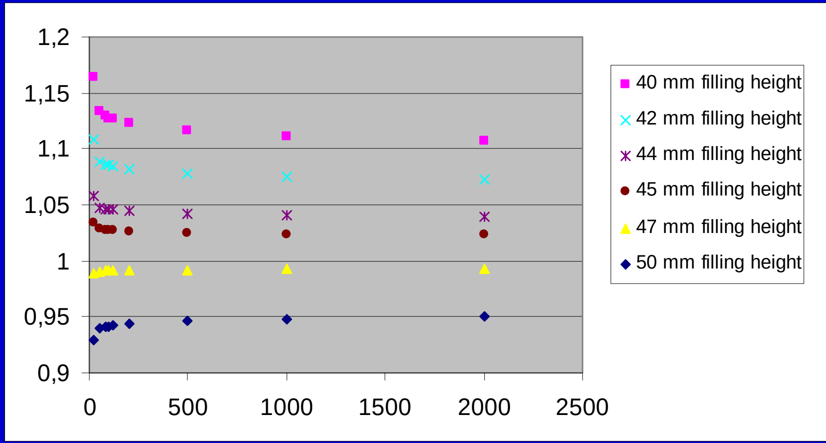
Influence of the filling height

Plastic vial filled with HCl 1N - Reference height=46,5 mm – diameter=39 mm

At 10 cm



At contact



About 5 % variation for 10% change in filling height

More important when the height is reduced

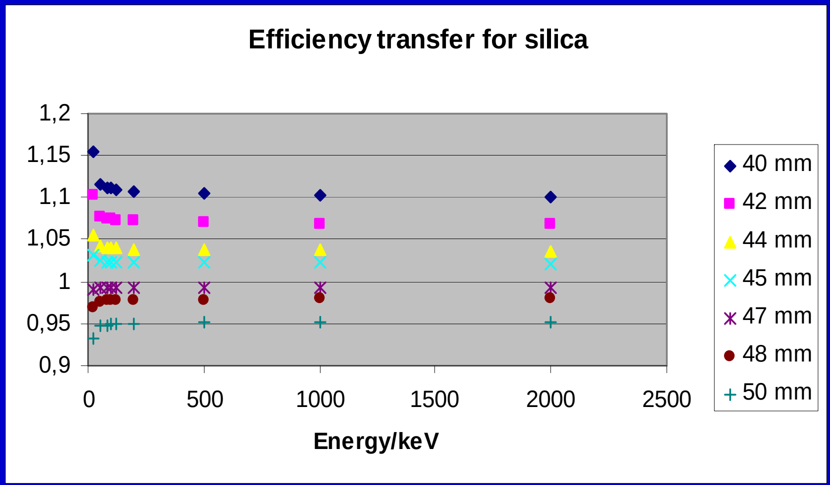
More sensitive for low-energies

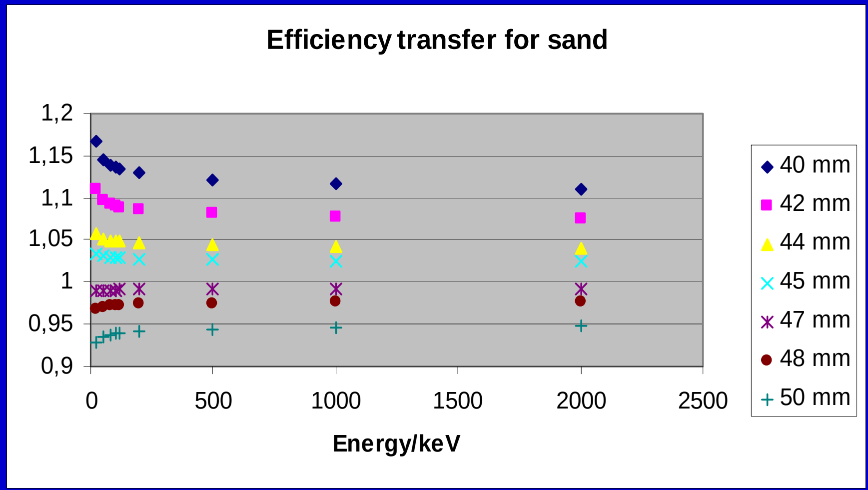
More sensitive at short source-to-detector distance

Influence of the filling height

Plastic vial at contact - Reference height=46,5 mm – diameter=39 mm

Silica (d=0.24) Sand-resin (d=1.54)





40 mm – 500 keV: HCl 1.11; silica 1.10; sand 1.12

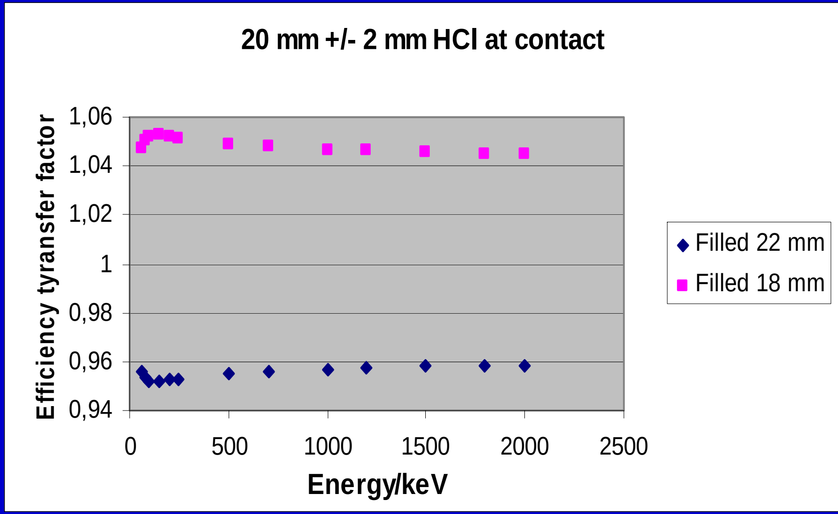
50 mm – 500 keV: HCl 0.946; silica 0.951; sand 0.944

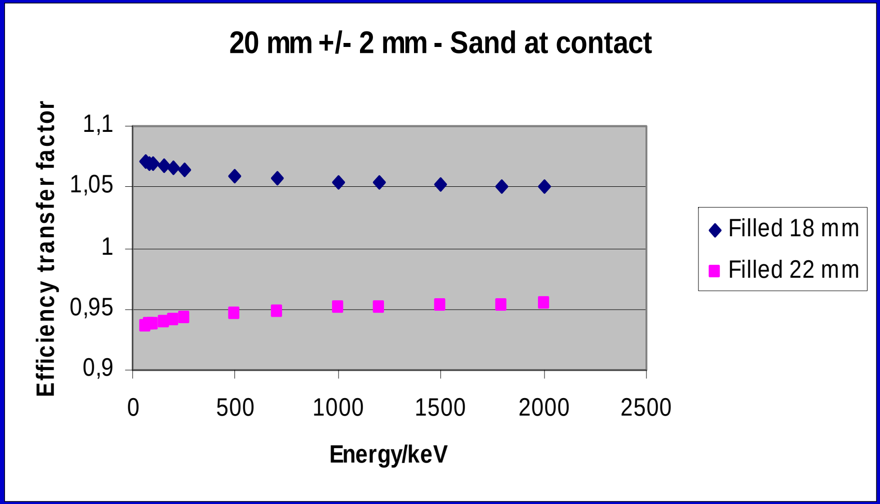
Influence of the filling height

Plastic vial at contact - Reference height=20 mm – diameter=39 mm

HCl (d=1.016)

Sand-resin (d=2.54)

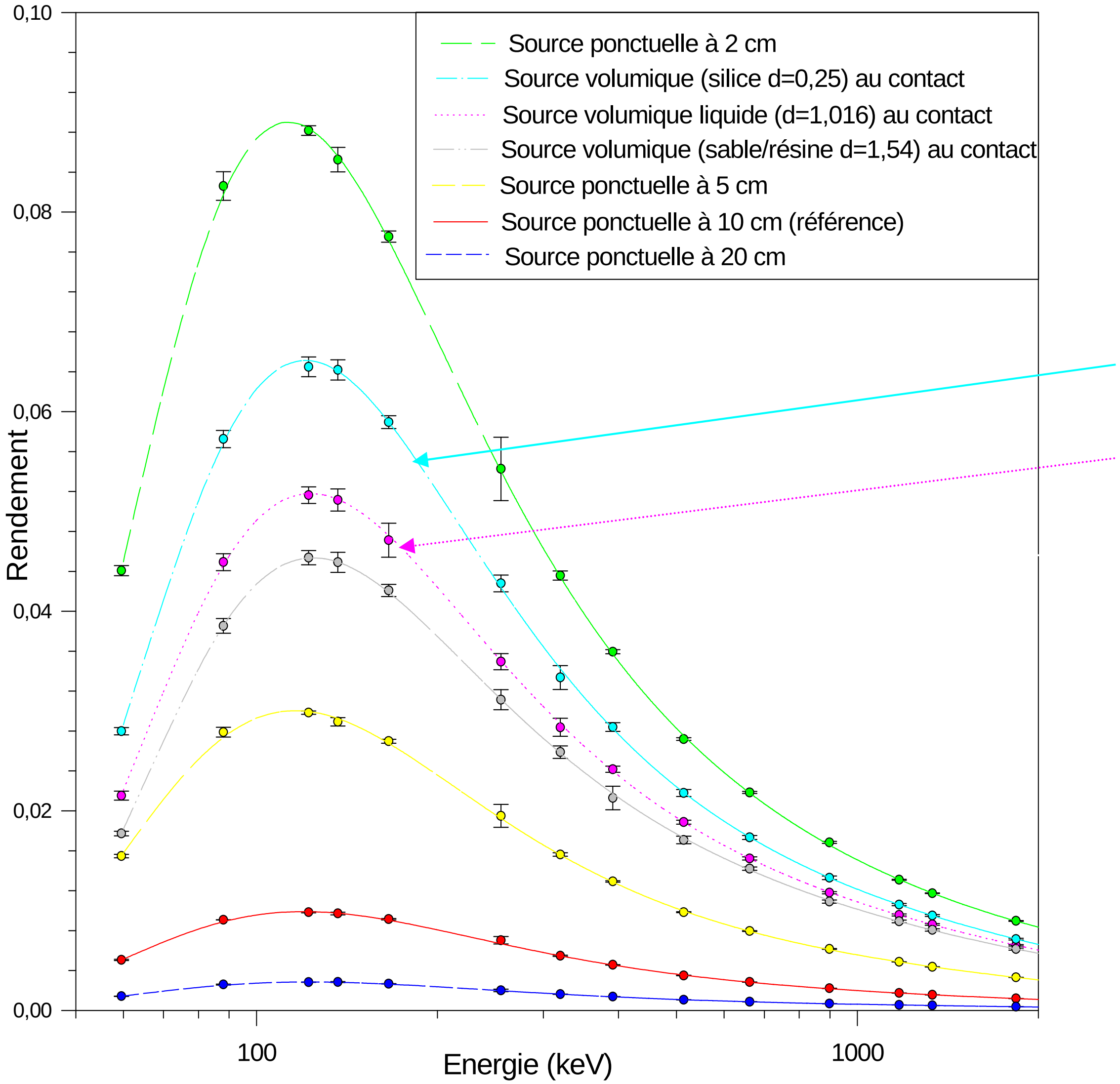




18 mm – 500 keV: HCl 1.049; sand 1.059

22 mm – 500 keV: HCl 0.955; sand 0.947

Efficiency calibration for different geometries



Experimental calibration with volume sources

Silica low density (d=0.24)

HCl 1N (d=1.016)

Sand-resin (d=1.54)

Efficiency transfer

Transfer from an efficiency calibration established with a liquid source (filled with 4.65 cm HCl) for matrixes silica and sand:

Densities:
- water/HCl = 1.016
- silica = 0.25
- sand/resin = 1.54

$$\varepsilon_{mes} = \varepsilon_{cal} \cdot f_{Self} = \varepsilon_{cal} \cdot \frac{\left[C_{att}\right]_{mes}}{\left[C_{att}\right]_{cal}}$$

Simple expression :

$$f_{Self} = \frac{\left[C_{att}\right]_{mes}}{\left[C_{att}\right]_{cal}} = \frac{\left[\frac{1-\exp(-\mu x)}{\mu x}\right]_{mes}}{\left[\frac{1-\exp(-\mu x)}{\mu x}\right]_{cal}}$$

Energy			
HCl	µ	Catt	
20	0,971	0,219	
50	0,225	0,621	
80	0,181	0,676	
100	0,169	0,693	
200	0,137	0,739	
500	0,098	0,803	
1000	0,072	0,851	
2000	0,050	0,892	
Silica	µ (cm^{-1})	Catt	f self
20	0,574	0,349	1,593
50	0,067	0,858	1,383
80	0,043	0,907	1,341
100	0,038	0,917	1,323
200	0,030	0,934	1,264
500	0,021	0,953	1,186
1000	0,015	0,965	1,135
2000	0,011	0,975	1,093
Sand	µ (cm-1)	Catt	f self
20	2,510	0,086	0,391
50	0,385	0,465	0,750
80	0,274	0,565	0,836
100	0,194	0,658	0,950
200	0,199	0,653	0,883
500	0,141	0,733	0,913
1000	0,103	0,794	0,934
2000	0,072	0,849	0,952

Application example

Transfer from an efficiency calibration established with a liquid source (filled with 4.65 cm HCl) for matrixes silica and sand:

$$f_{Self} = \frac{[C_{att}]_{mes}}{[C_{att}]_{cal}}$$

Comparison
of the generalised formula
(ETNA code) with the
simple one

Energy	Silica	Silica	Simple	Complete
	0,24	mu	Transfer	ETNA
50	0,281	0,0674	1,383	
60				1,296
80	0,178	0,0427	1,341	1,270
100	0,158	0,0379	1,323	1,251
200	0,123	0,0295	1,264	1,201
500	0,087	0,0209	1,186	1,142
1000	0,064	0,0153	1,135	1,103
2000	0,045	0,0107	1,093	1,072
	Sand	Sand	Simple	ETNA
	1,54	mu	Transfer	ETNA
50	0,25	0,385	0,750	
60				0,828
80	0,178	0,274	0,836	0,865
100	0,126	0,194	0,950	0,882
200	0,129	0,199	0,883	0,908
500	0,092	0,141	0,913	0,932
1000	0,067	0,103	0,934	0,948
2000	0,047	0,072	0,952	0,963

Summary

- Self attenuation is of main importance for low energies and high densities
- In case of high attenuation only a thin layer of the sample located close to the detector is important
- In case of homogeneous matrix, it can be computed if the attenuation cœfficient is known

Methods for self-attenuation correction

- Empirical methods – simplified computing
- Analytic approach
- Monte Carlo methods