Detection efficiency

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Detector efficiency

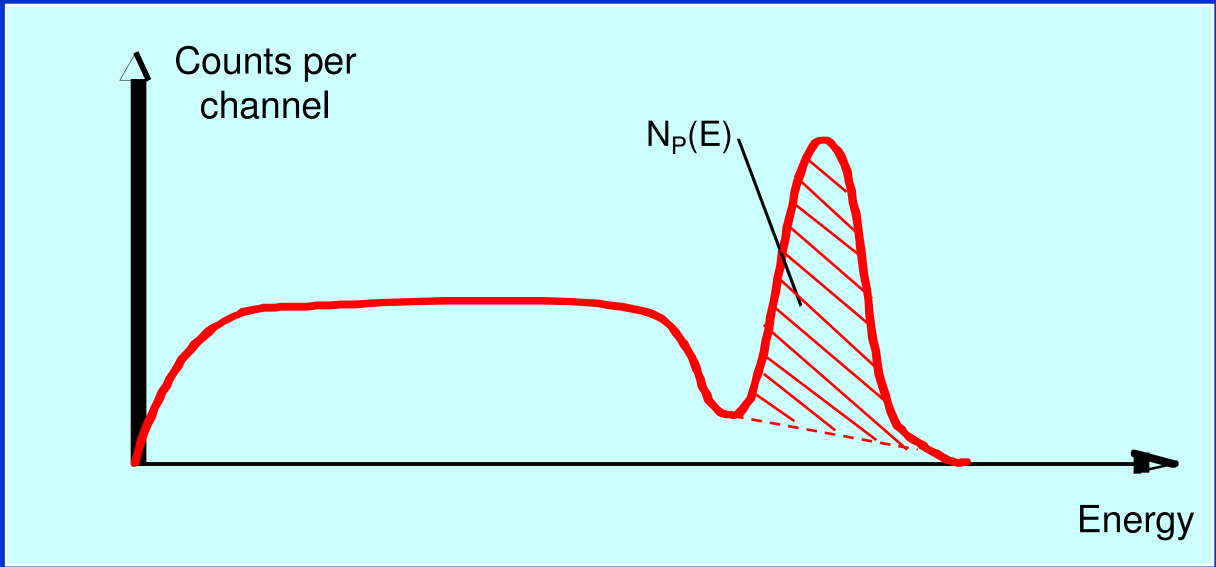
- Full energy peak efficiency
  - Definition
  - Experimental calibration
  - Curve fitting
- Total efficiency
  - Definition
  - Experimental calibration
- Monte Carlo simulation

FULL ENERGY PEAK EFFICIENCY

Full-energy Peak Efficiency (FEPE): $\varepsilon_P(E)$

Ratio of the number of counts in full-energy peak corresponding to energy E ($N_P(E)$), by the number of photons with energy E emited by the source ($F(E)$)

$$\varepsilon_P(E) = \frac{N_P(E)}{F(E)}$$



$\varepsilon_P(E)$ depends on the source-detector geometry and on the energy

Full-energy Peak Efficiency (FEPE): $\varepsilon_P(E)$

$\varepsilon_p(E)$ depends on the geometrical conditions and on the energy

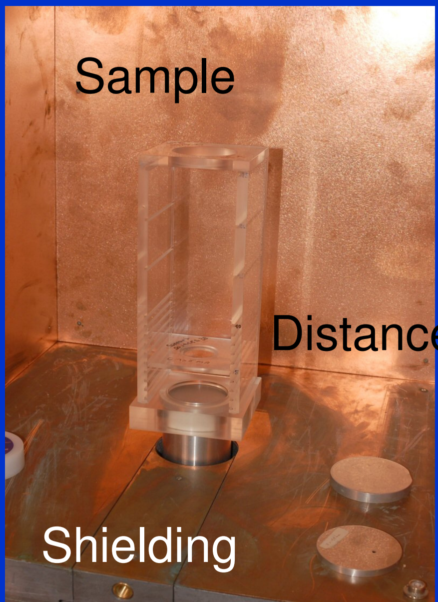
Measurement conditions

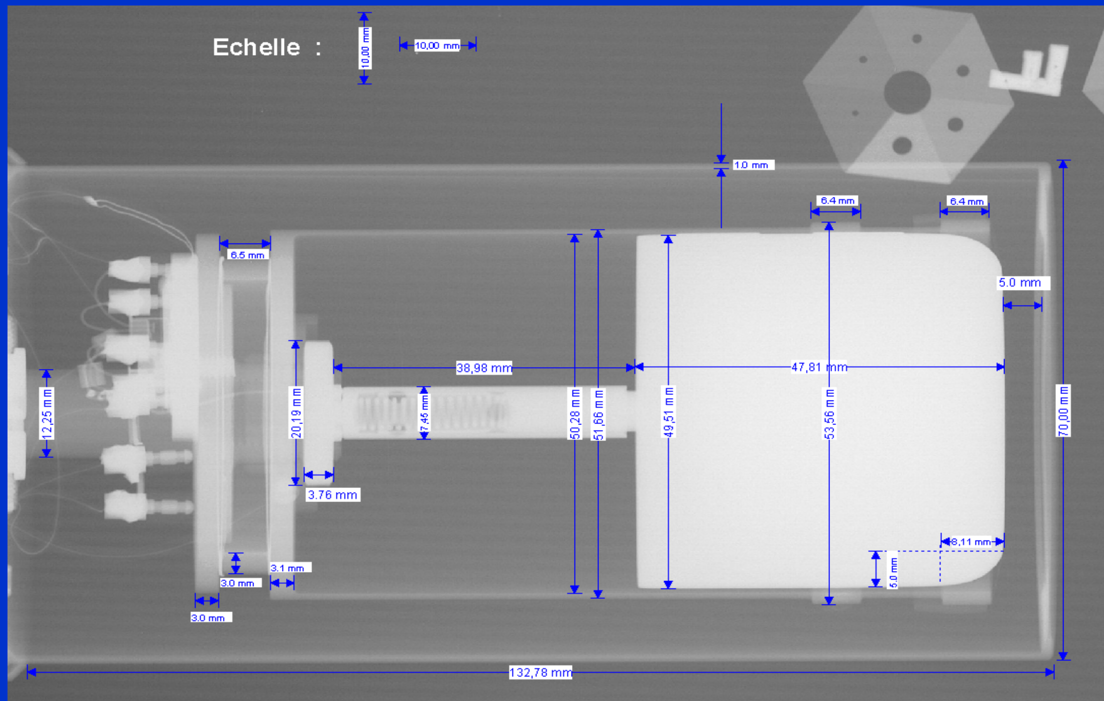
Detector characteristics

$$\varepsilon_P(E) = \varepsilon_G \cdot \varepsilon_I(E)$$

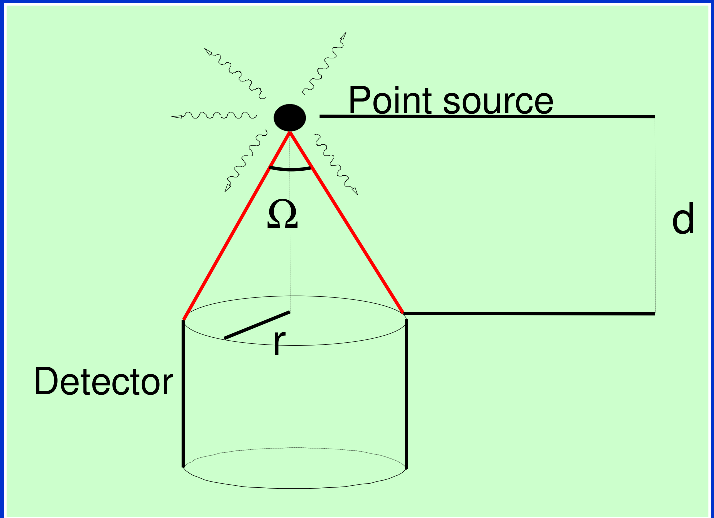
Intrinsic efficiency

Geometrical efficiency





Geometrical efficiency



Ω = solid angle between source and detector (*sr*)

For a point source :

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2 + r^2}}\right)$$

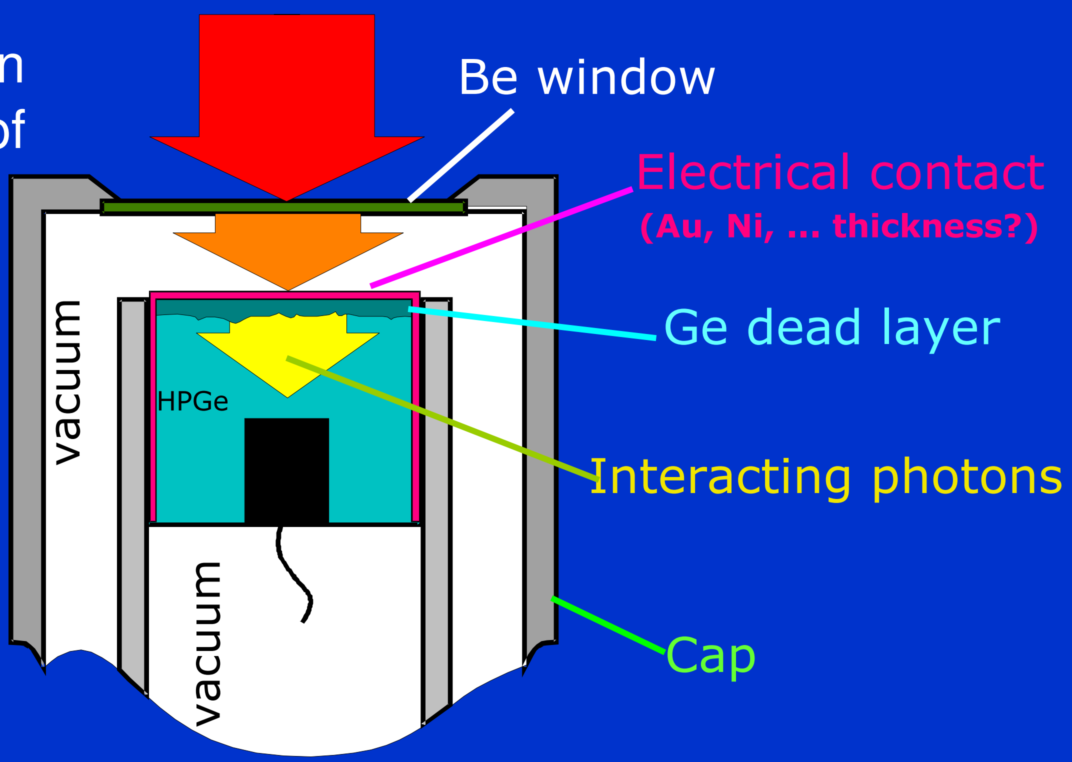
Ratio of the number of photons emitted towards the detector by the number of photons emited by the source

$$\varepsilon_G = \frac{\Omega}{4\pi}$$

ε_G depends only on the source-detector geometry

Intrinsic efficiency

$\varepsilon_I(E)$ = Ratio of the number of counts in full-energy peak by the number of impiging photons



$\varepsilon_I(E)$ depends on the energy of the incident photons:

transmission

absorption

full-energy deposition

Difficulty: exact composition badly known

Calculation of the detector FEP efficiency

Transmission probability through material i

with thickness x_i : (Beer-Lambert law) : ⇨ $P_T(E, x_i) = \exp(-\mu_i(E) \cdot x_i)$

Thus

Interaction probability in the same material: ⇨ $P_I(E, x_i) = 1 - P_T(E, x_i)$

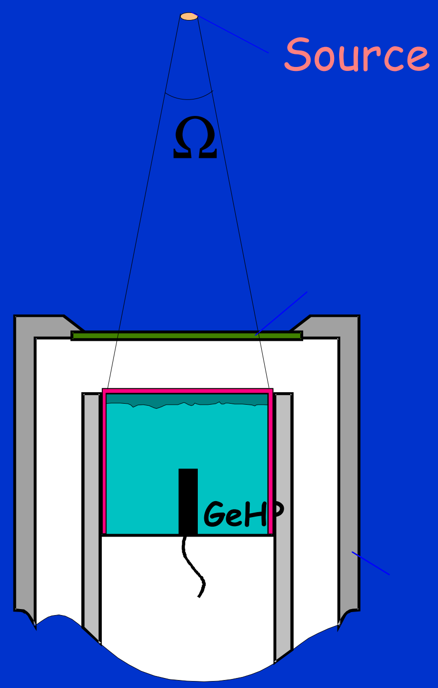
To result in a count in the FEP peak:

The photon must :

- be emitted in the Ω solid angle,

- cross the screens (air,n window, dead layer,…) without being absorbed,

- and be totally absorbed in the detector active volume.



Calculation of the detector efficiency

Transmission through screens

Interaction in the detectorr volume

$$R^P(E) = \int_{\Omega} \prod_i \left(\exp(-\mu_i(E)\cdot x_i)\right)\cdot\left(1-\exp(-\mu_d(E)\cdot x_d\right)\cdot P_P(E)\cdot d\Omega$$

Probability of total absorption in the detector

- For the low-energy range: $P_p(E) \approx \tau_d(E)/\mu_d(E)$

- For higher energies:

  Total absorption is due to succesive effects : multiple scattering

  Thus the calcul is not possible

Calculation of the detector efficiency

Many difficulties for an accurate calculation

- Exact knowledge of the detector parameters:

  materials *(composition)*

  geometry *(thickness, position …)*

- Data used in the calculation :

  Attenuation coefficients

  Material density

- Semi-conductor effects: parameters and physical interaction :

  Electrical field

  Electrodes

Radiography of a HPGe detector:
Rounded crystal, axially shifted, tilted …

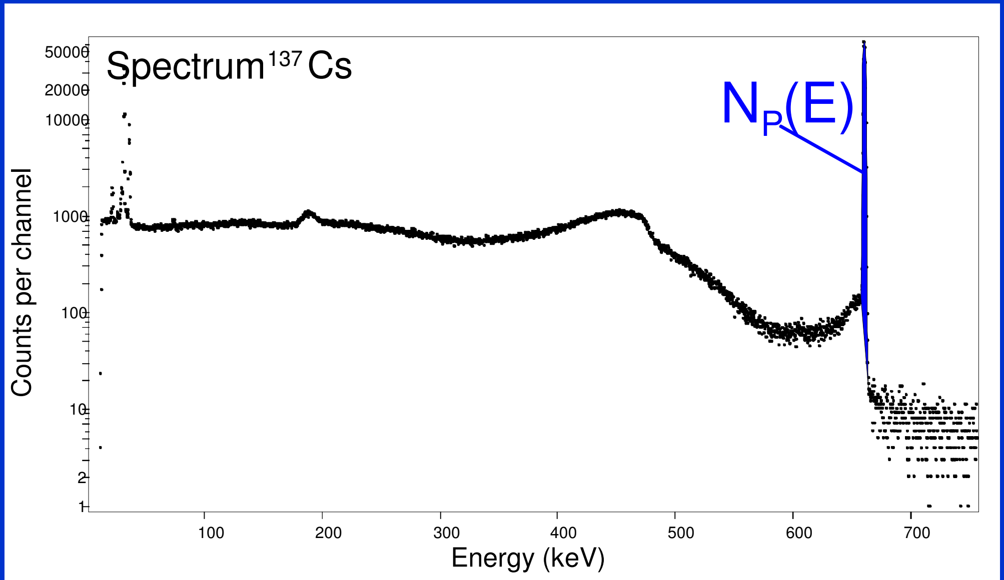
Experimental FEP efficiency calibration

$$\varepsilon_P(E) = \frac{N_P(E)}{F(E)}$$

$$\varepsilon_P(E) = \frac{N_P(E)}{A \cdot I_\gamma(E)}$$

This is performed using standard radionuclides with standardized activity A (Bq) with photon emission intensities, Iγ well known

$N_P(E)$: peak net area



$\varepsilon_P(E)$ Full-energy peak (FEP) efficiency depends on the energy and on the source-detector geometrical arrrangement

Associated standard uncertainty

Efficiency calibration

$$\varepsilon(\mathrm{E}) = \frac{\mathrm{N(E)}}{\mathrm{A} \cdot \mathrm{I}_\gamma(\mathrm{E})}$$

$$\left(\frac{\Delta\varepsilon(\mathrm{E})}{\varepsilon}\right)^2 = \left(\frac{\Delta\mathrm{N(E)}}{\mathrm{N}}\right)^2 + \left(\frac{\Delta\mathrm{A}}{\mathrm{A}}\right)^2 + \left(\frac{\Delta\mathrm{I(E)}}{\mathrm{I(E)}}\right)^2$$

$$\frac{\Delta N(E)}{N(E)} = \frac{\sqrt{N(E)}}{N(E)} = \frac{1}{\sqrt{N(E)}} \qquad \frac{\Delta A}{A} = 5 \cdot 10^{-3} \qquad \frac{\Delta I_\gamma(E)}{I_\gamma(E)} = 1 \cdot 10^{-3}$$

Influence of the peak area :

if N = 10^4 $\Delta N/N = 10^{-2}$ -> $\Delta\varepsilon/\varepsilon = 1{,}1\ 10^{-2}$

if N = 10^5 $\Delta N/N = 3{,}1\ 10^{-3}$ -> $\Delta\varepsilon/\varepsilon = 6\ 10^{-3}$

FEP efficiency calibration

To get an efficiency values at any energy : energy calibration over the whole energy range

1. Use different radionuclides to get energies regularly spaced over the range of interest

   Single gamma-ray emitters : ^{51}Cr (320 keV), ^{137}Cs (662 keV)

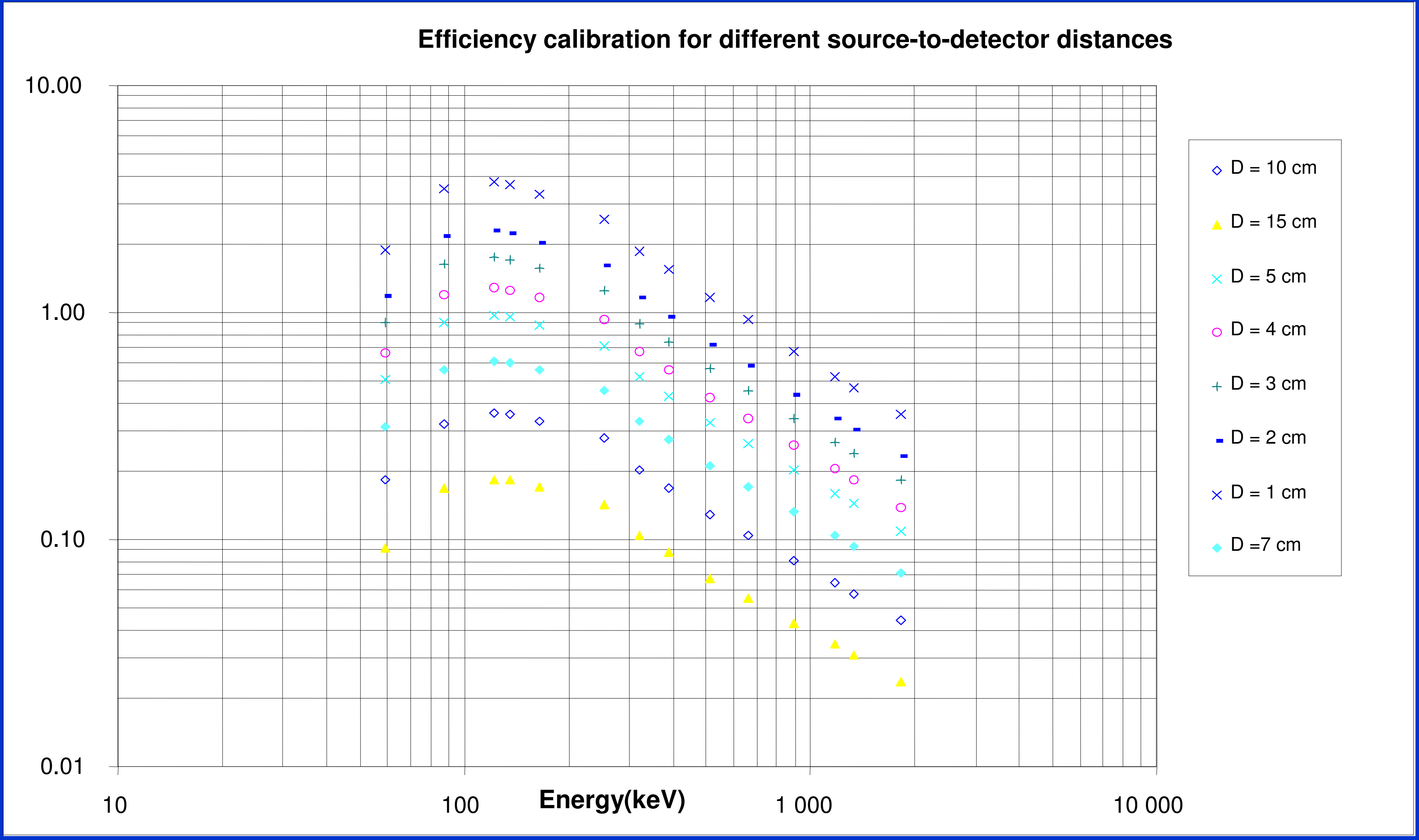
   ^{54}Mn (834 keV) : one efficiency value per one measurement

   Multigamma emitters : ^{60}Co, ^{133}Ba, ^{152}Eu, ^{56}Co : several efficiencies values per one measurement , but coincidence summing effects !

For each energy, discrete values of the FEP efficiency $\varepsilon(E_1)$, $\varepsilon(E_2)$, … $\varepsilon(E_n)$

2. Computation of the efficiency for
   1. Local interpolation
   2. Fitting a mathematical function to the experimental values

FEP efficiency calibration



Efficiency calibration - Interpolation

Local interpolation

$E_1 \rightarrow \varepsilon_1$

$E_2 \rightarrow \varepsilon_2$

To get $\varepsilon(E)$ for $E_1 < E < E_2$

Solution 1 : linear interpolation →

$$\varepsilon(E) = \varepsilon_1 + \frac{(E - E_1)}{(E_2 - E_1)} \cdot (\varepsilon_2 - \varepsilon_1)$$

! : valid only for close energies

Solution 2 : logarithmic interpolation

$$ln(\varepsilon(E)) = ln(\varepsilon_1) + \frac{(ln\,E - ln\,E_1)}{(ln\,E_2 - ln\,E_1)} \cdot (ln\,\varepsilon_2 - ln\,\varepsilon_1)$$

Efficiency calibration - Interpolation

Local interpolation : example

Determine efficiency for E = 662 keV (^{137}Cs) knowing :

1. E_1 = 569 keV (^{134}Cs) $\varepsilon_1 = 2.21\ 10^{-3}$
   E_2 = 766 keV (^{95}Nb) $\varepsilon_2 = 1.67\ 10^{-3}$

   Linear interpolation : $\varepsilon(E) = 1.96\ 10^{-3}$
   Logarithmic interpolation : $\varepsilon(E) = 1.92\ 10^{-3}$

2. E_1 = 122 keV (^{57}Co) $\varepsilon_1 = 8.22\ 10^{-3}$
   E_2 = 1 173 keV (^{60}Co) $\varepsilon_2 = 1.13\ 10^{-3}$

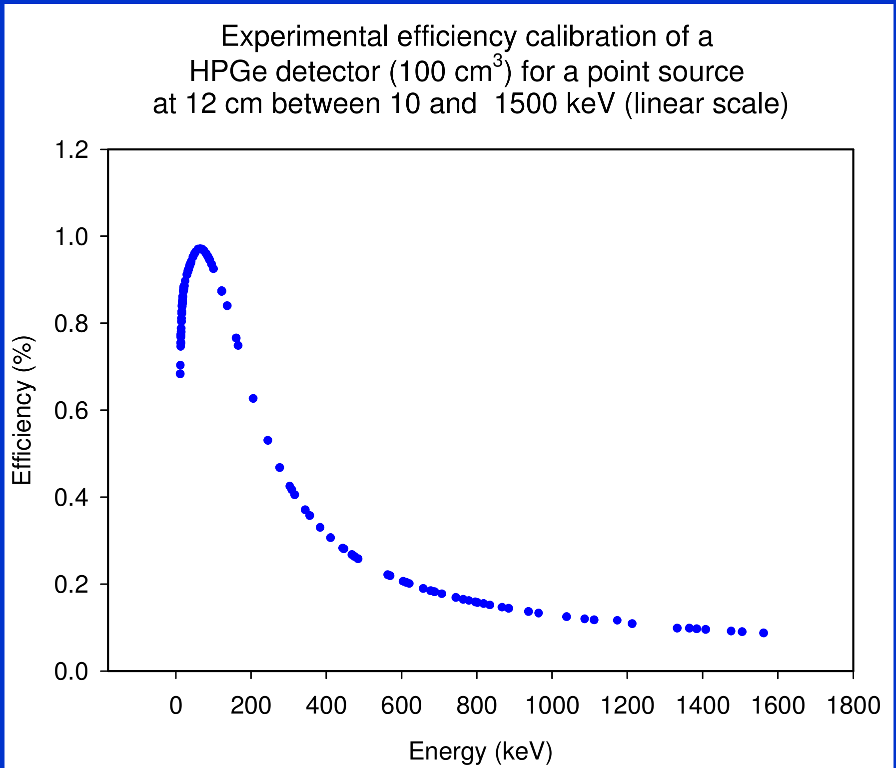
   LInear interpolation *:* ε*(E) = 4.57 10^{-3}*
   Logarithmic interpolation : $\varepsilon\ (E) = 1.87\ 10^{-3}$

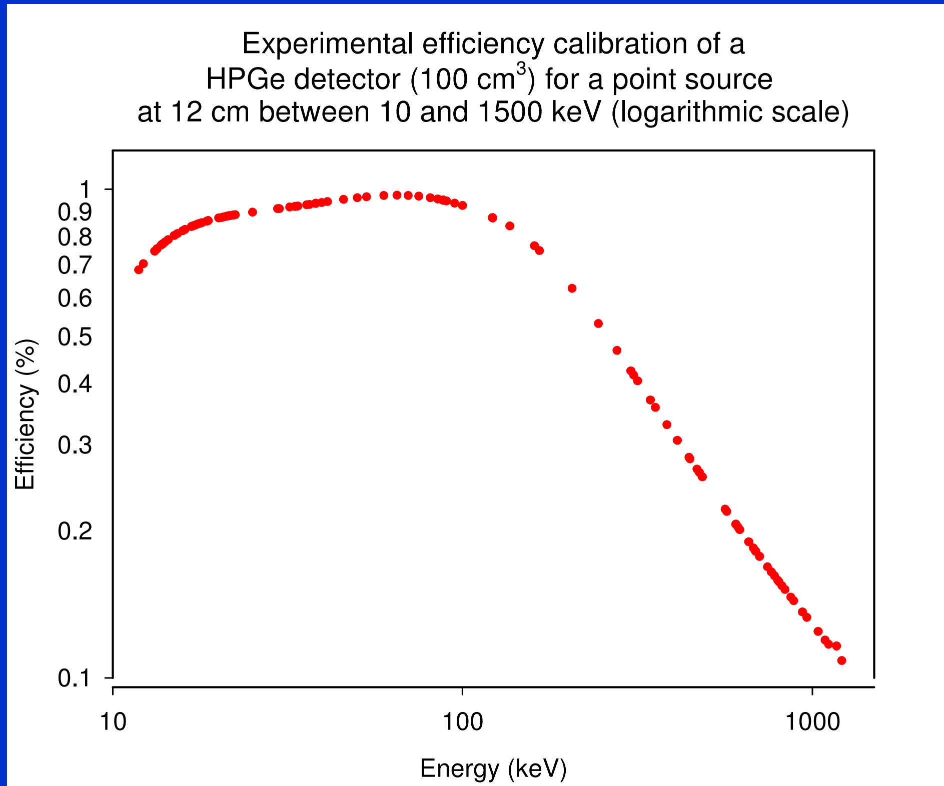
Actual value (at 662 keV) : $\varepsilon(E) = 1.90\ 10^{-3}$

Conclusion : approximation to be used only if high uncertainties (> 10 %) are acceptable

Efficiency calibration : mathematical fitting (1)

Determination of the best fitted function to a given set of experimental data (energy, efficiency)





In the logarithmic scale , the shape is smoother than in the linear scale.

Efficiency calibration : mathematical fitting

Functions frequently used:

Polynomial fitting in the log-log scale:

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot (\ln E)^i$$

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot E^{-i}$$

Remarks :

-a_i coefficients are determined using a least-squares fitting method

- experimental data must be weighted

- the polynomial degree (n) must be adjusted depending on the number of experimental data (p) : n << p

- in some case two different functions can be used with a cross point

- check the resulting fitted curves !

Efficiency calibration : mathematical fitting

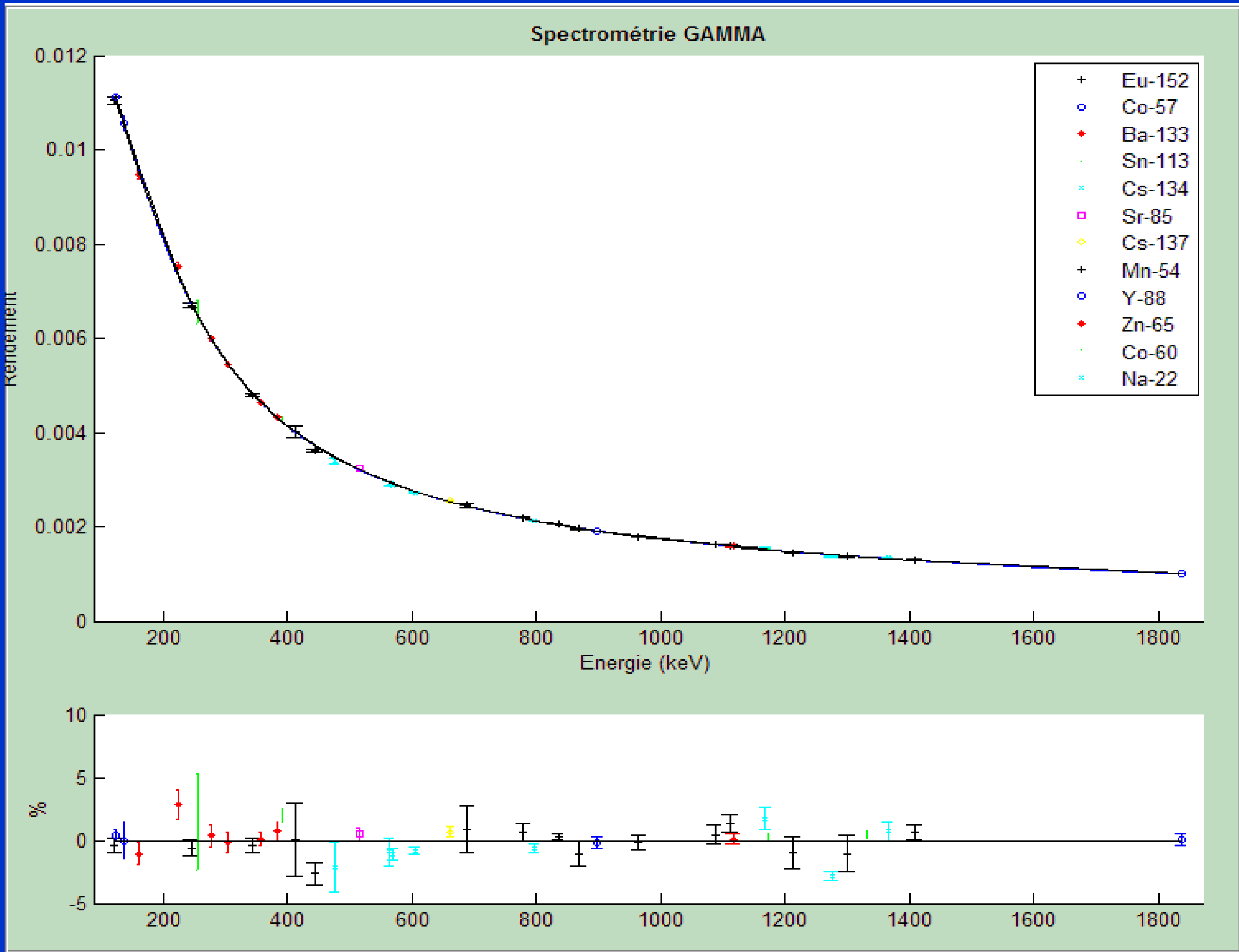
Example : 40 experimental values in the 122-to-1836 keV range

Fitting function :

$$\ln \varepsilon(E) = \sum_{i=0}^{n} a_i \cdot (\ln E)^i$$

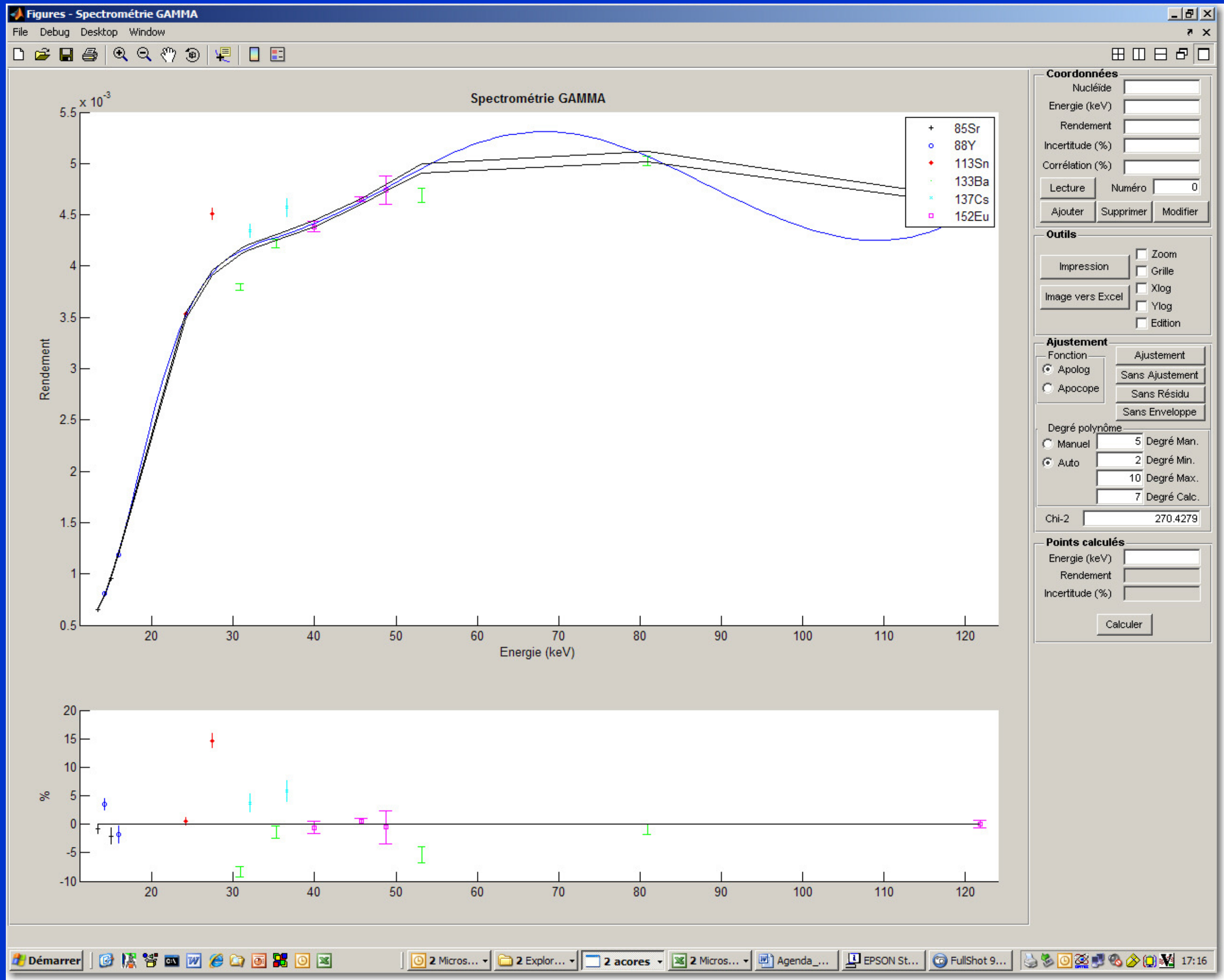
Adjusted coefficients :

fitting	122 to 1836 keV
deg0	-34,11961
deg1	48,16797
deg2	-25,89215
deg3	5,80219
deg4	-0,40503
deg5	-0,01632



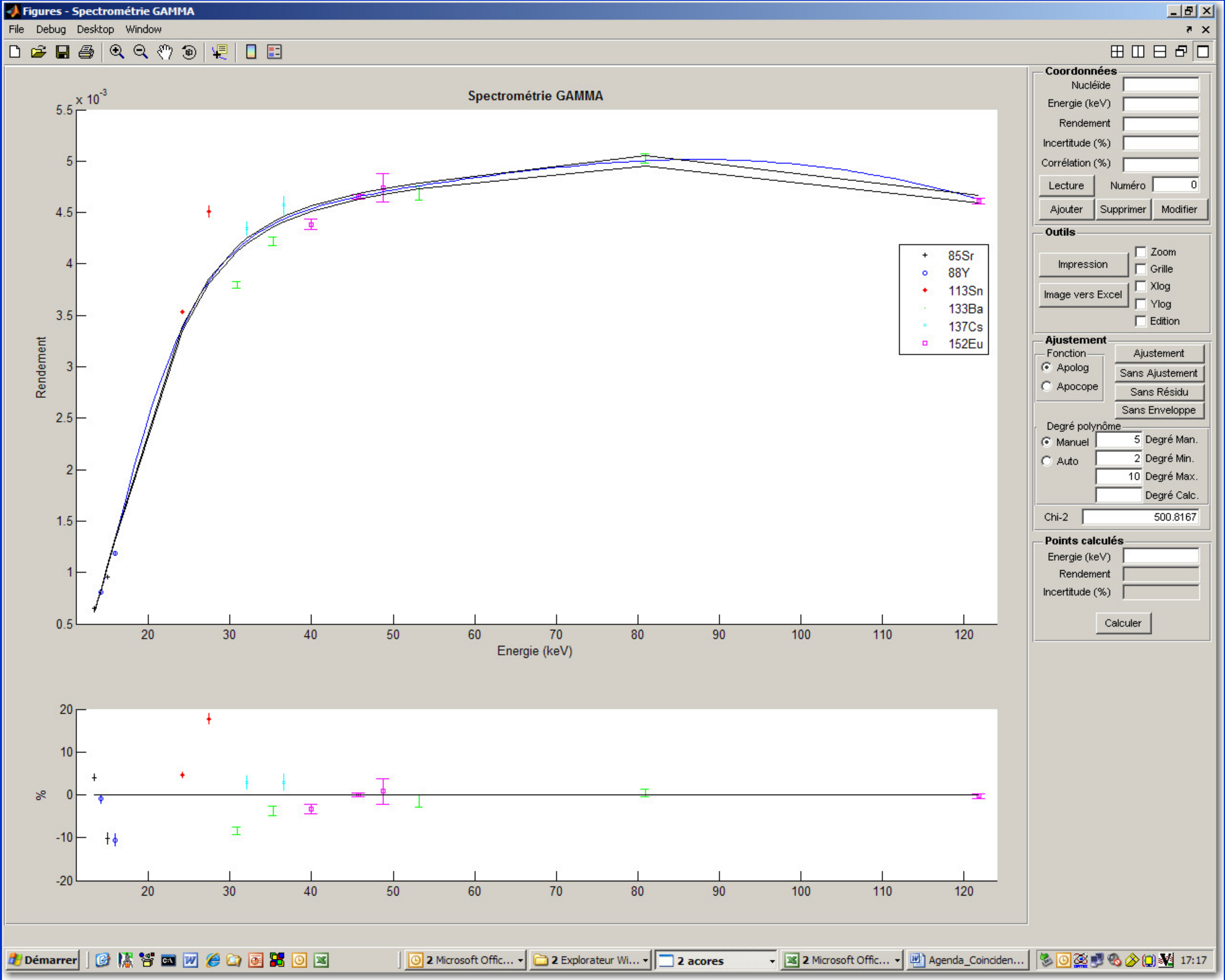
Efficiency calibration : mathematical fitting

Efficiency fitting must be visually checked



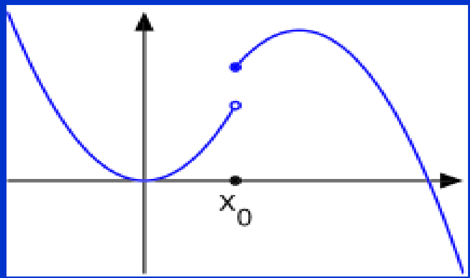
In the case of cross points be carefull :

- Avoid zones with important inflexion
- Avoid high degree polynomials



Spline functions

- Simply a curve
- Special function defined piecewise by polynomials

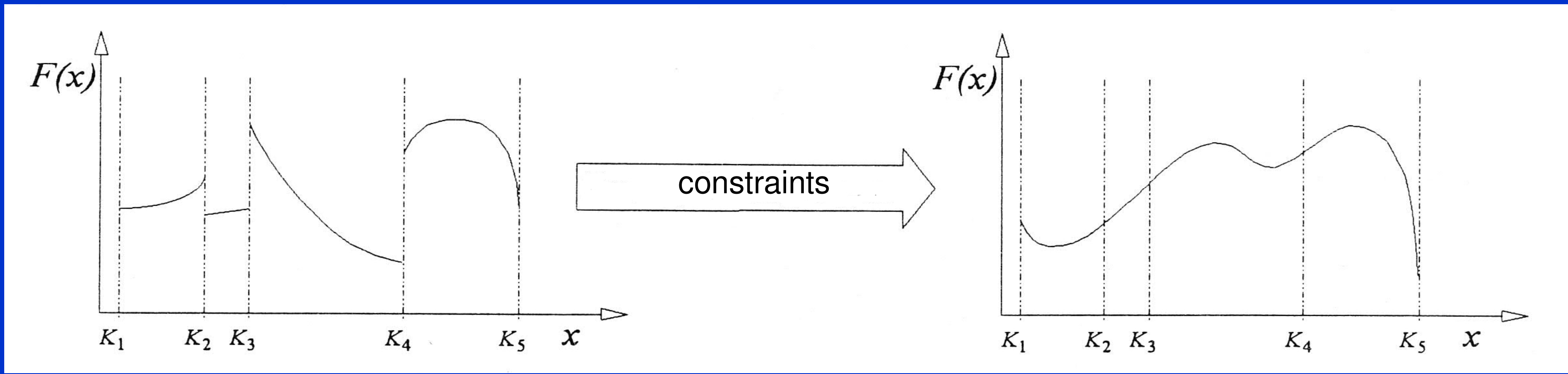


- A piecewise polynomial f(x) is obtained by dividing of X into contiguous intervals, and representing f(x) by a separate polynomial in each interval

- The polynomials are joined together at the interval endpoints (knots) in such a way that a certain degree of smoothness of the resulting function is guaranteed

Slides from Dr. Oliver Ott - PTB

Piecewise defined polynomials



$\{K_i\}$ (i = 1, …, k+1)

$K_1 \leq x_{min} < K_2 < \ldots < K_k < x_{max} < K_{k+1}$

Series of knots

x_{min}: minimum x-value of the data points

x_{max}: maximum x-value of the data points

- 2 consecutive knots establish an interval $[K_i, K_{i+1})$ where a polynomial $P_i(x)$ of order n (degree n-1) is defined.
- In total k intervals are fixed by the knots.
- Outside the interval i the polynomial $P_i(x)$ is not defined.
- At the joining points $K_2 \ldots K_k$ they have to fit smoothly – continous up to the (n-2) derivative

Spline functions mathematically

k polynomials in k intervals: $$F(x) = \sum_{i=1}^{k} P_i(x)$$

where

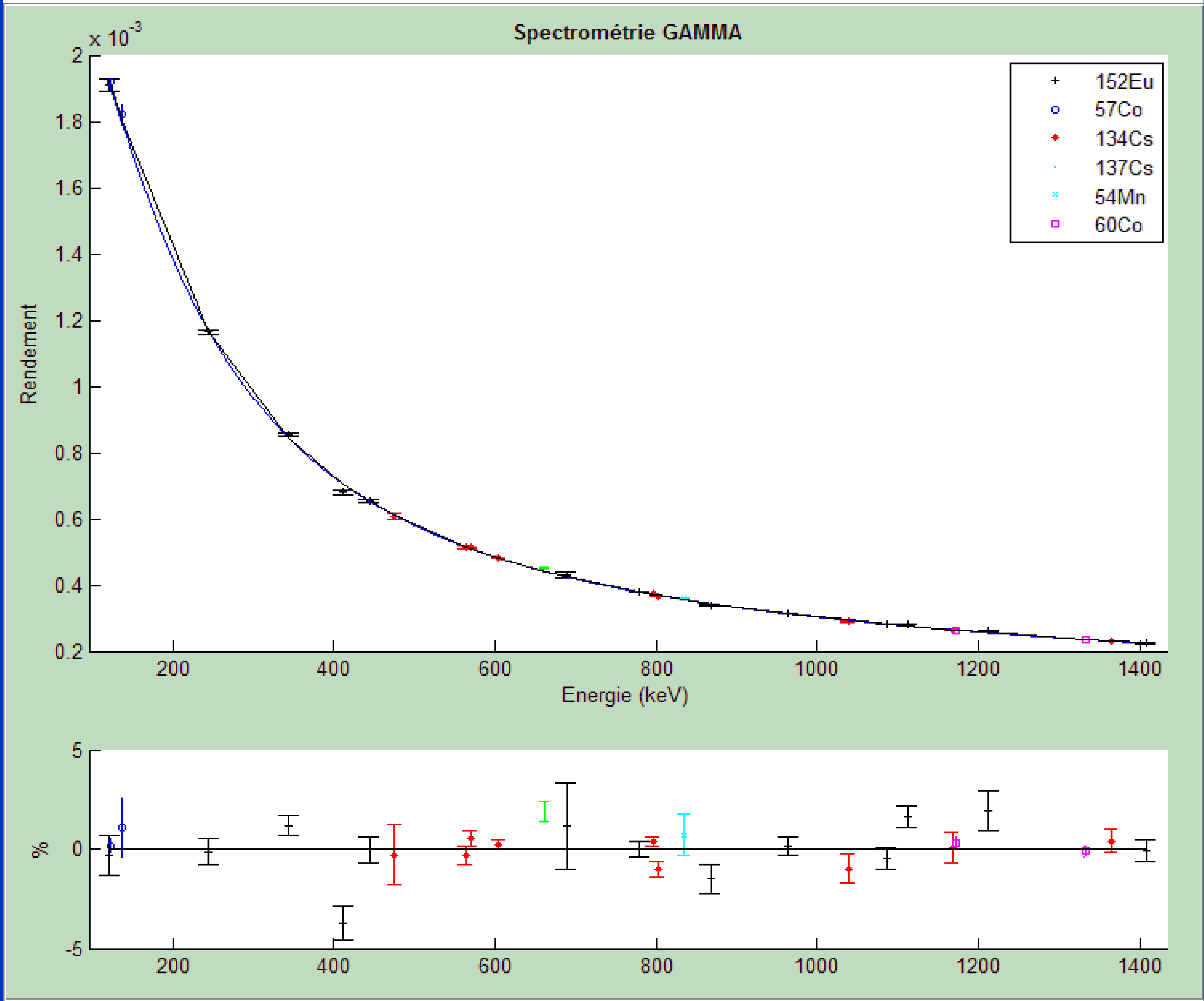
$$P(x) = \begin{cases} \sum_{j=1}^{n} a_{ij} \cdot (x - K_i)^{j-1} & \text{for } K_i \leq x \leq K_{i+1} \\ 0 & \text{else} \end{cases}$$

Condition that a spline function solution must satisfy:

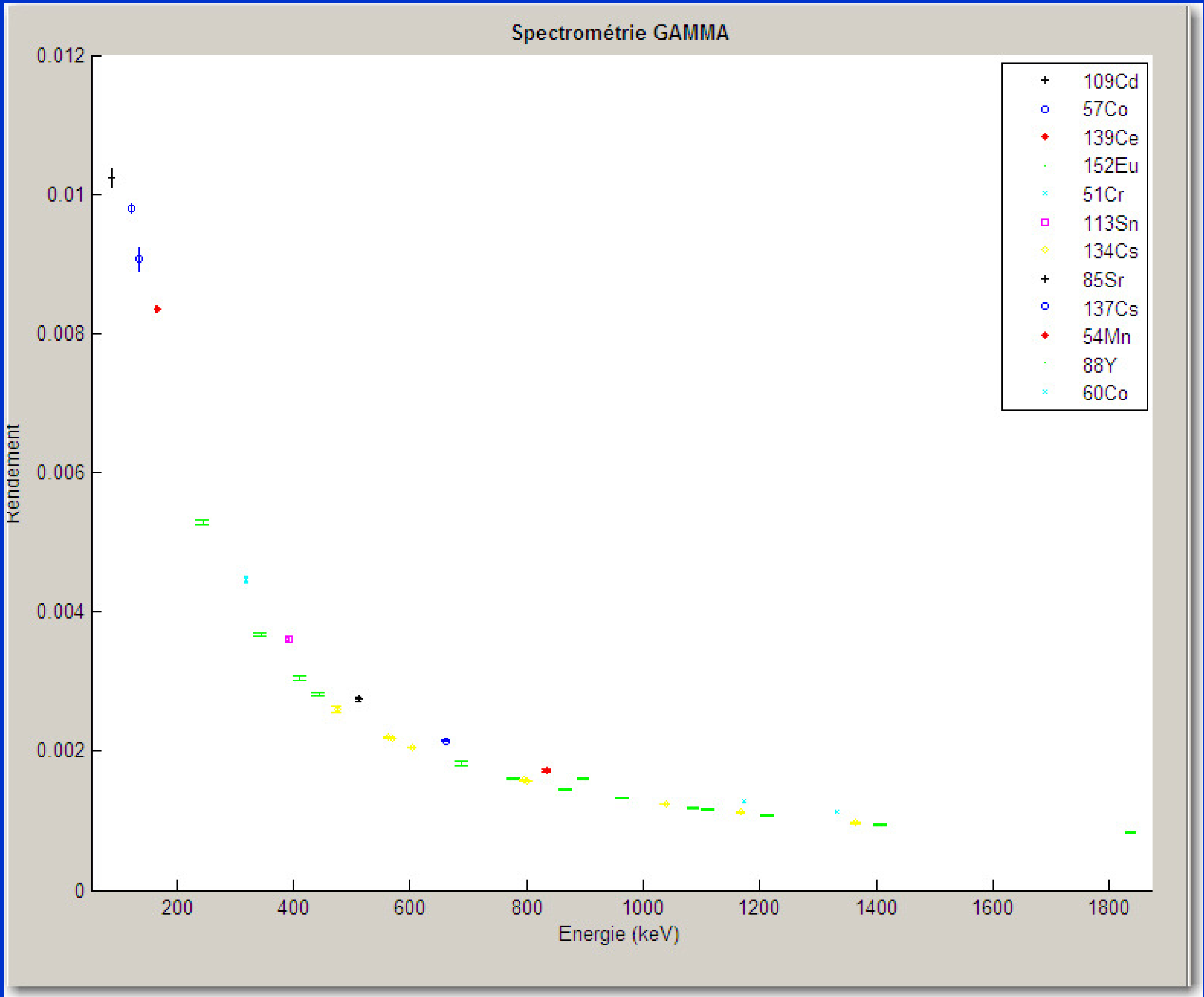
For the derivatons of neighboured polynomials at all knots:

$$P_i^{(m)}(K_{i+1}) - P_{i+1}^{(m)}(K_{i+1}) = 0, \quad 0 \leq m \leq n - 2$$

Efficiency calibration at 25 cm



Efficiency calibration at 10 cm



Multigamma emitters -> Coincidence summing effets

FEP Efficiency calibration : remarks

- Efficiency calibration for reference geometry
  - For point source : relative uncertainty 1-2 %

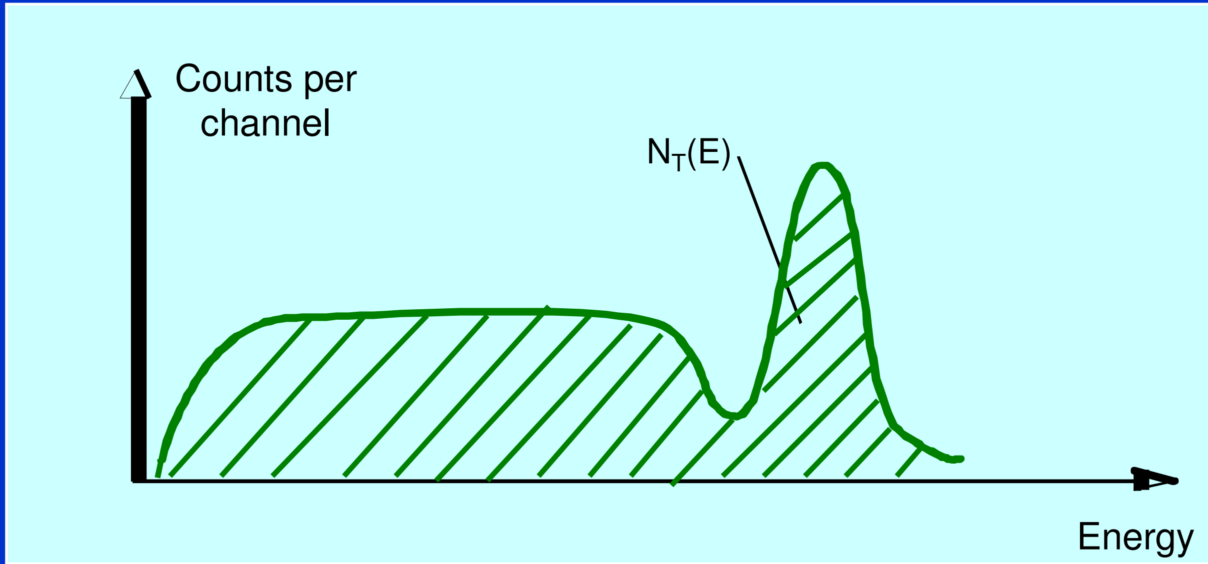
- Corrective factors needed if different measurement geometry

TOTAL EFFICIENCY

Total Efficiency (TE): $\varepsilon_T(E)$

Ratio of the total number of counts in the spectrum $(N_T(E))$, by the number of photons with energy E emitted by the source ($F(E)$)

$$\varepsilon_T(E) = \frac{N_T(E)}{F(E)}$$



$\varepsilon_T(E)$ depends on the source-detector geometry

and on the energy

Calculation of the total efficiency

$$\varepsilon_T(E) = \int_\Omega \prod_i \left(\exp\left(-\mu_i(E)\cdot x_i\right)\right)\cdot\left(1-\exp\left(-\mu_d(E)\cdot x_d\right)\right)\cdot d\Omega$$

Attenuation in screens

Detector interaction

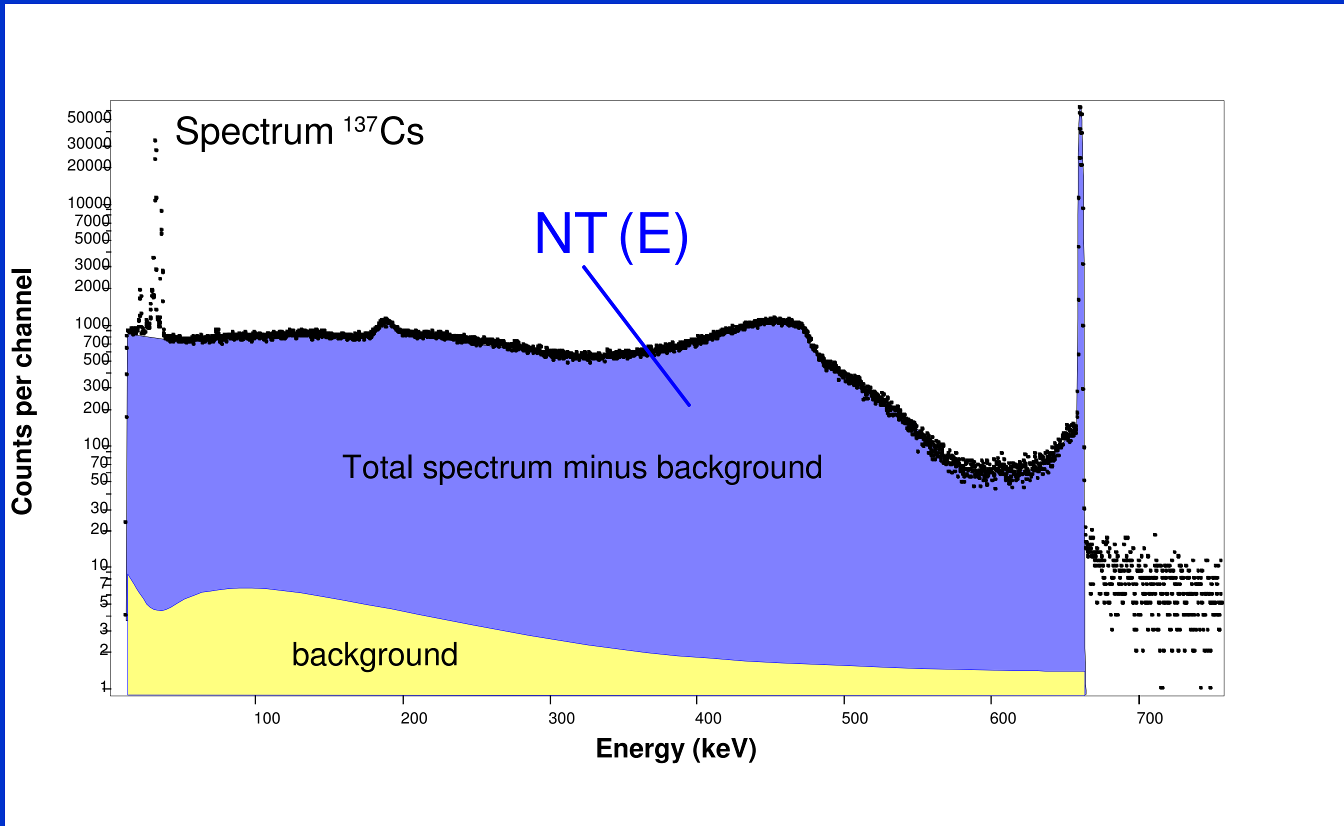
where
- x_i : i shield thickness
- x_d : detector thickness

Total efficiency depends on:

- Solid angle Ω
- Attenuations in absorbing layers (air, window, dead layer, etc)
- Interaction in the detector active volume (any effect)

And on the environment ! (scattering)

Experimental total efficiency calibration $\varepsilon_T(E)$



$$\varepsilon_T(E) = \frac{NT(E)}{A.I_\gamma(E)}$$

Experimental total efficiency calibration $\varepsilon_T(E)$

1. Using <u>mono-energetic</u> radionuclides

   ^{241}Am (60 keV), ^{109}Cd (88 keV), ^{139}Ce (166 keV-$T_{1/2}$=138 d), ^{51}Cr (320 keV- $T_{1/2}$= 28d), ^{85}Sr (514 keV- $T_{1/2}$=65 j), ^{137}Cs (662 keV), ^{54}Mn (834 keV - $T_{1/2}$=302 d)

2. For the low-energy range, if P-type detector, ^{57}Co can be used (mean energy and sum of emission intensities)

3. For the high energy range, ^{88}Y and ^{60}Co can be used

... But complex decay scheme

Approximation of the total efficiency with 2-photons emitters:

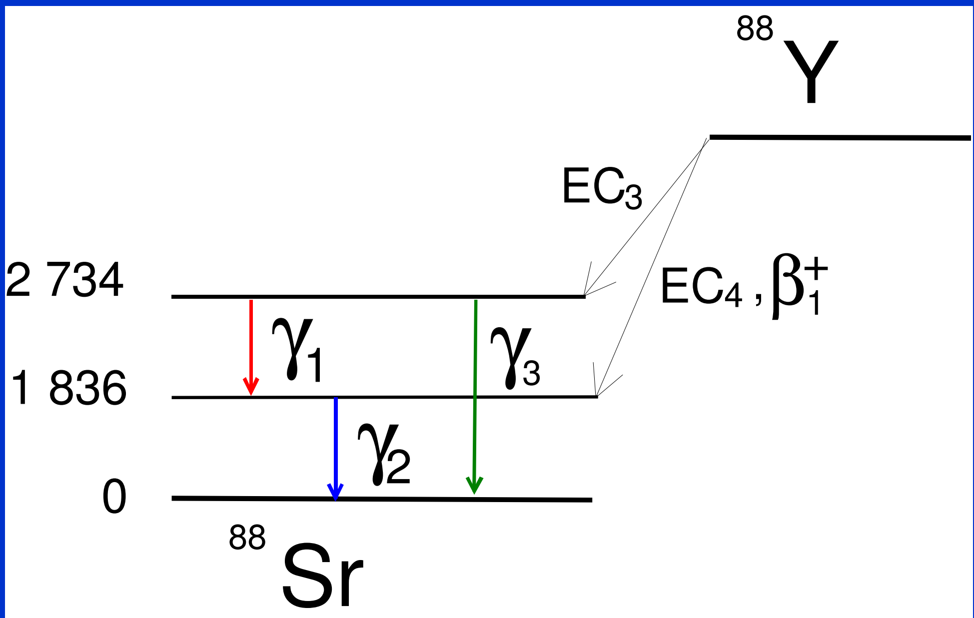
$$N_T = A\,(I_{\gamma 1} \cdot \varepsilon_{T\,1} + I_{\gamma 2} \cdot \varepsilon_{T\,2} - I_{\gamma 1} \cdot \varepsilon_{T\,1} \cdot P_{12} \cdot I_{\gamma 2} \cdot \varepsilon_{T\,2})$$

^{88}Y : E_1 =898 keV and E_2 = 1836 keV (or ^{65}Zn : 511 and 1115keV)

E_1 = 898 keV → ε_{T1} can be extrapolated from previous data

(^{54}Mn – 834 keV)

$P_{12} = 1$



$$\varepsilon_{T2} = \frac{N_T - A \cdot I\gamma_1 \cdot \varepsilon_{T1}}{A \cdot I_{\gamma 2} \cdot \left(1 - I_{\gamma 1} \cdot \varepsilon_{T1}\right)}$$

Approximation of the total efficiency with 2-photons emitters:

$$N_T = A\,(I_{\gamma 1} \cdot \varepsilon_{T1} + I_{\gamma 2} \cdot \varepsilon_{T2} - I_{\gamma 1} \cdot \varepsilon_{T1} \cdot P_{12} \cdot I_{\gamma 2} \cdot \varepsilon_{T2})$$

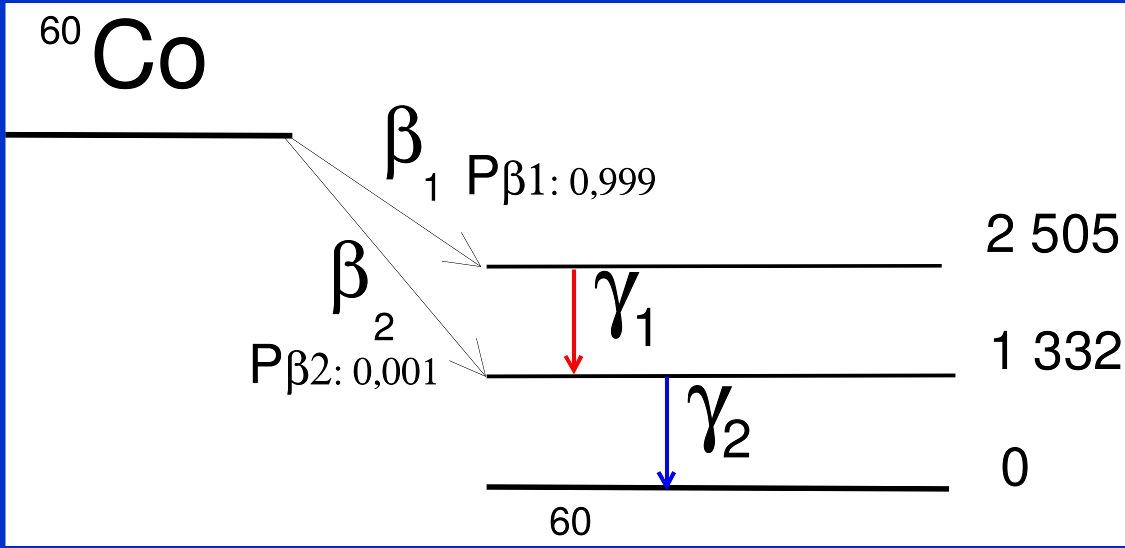
^{60}Co :

E_1 = 1 173 keV $I_{\gamma 1} = 0{,}99$

E_2 = 1 332 keV $I_{\gamma 2} = 0{,}98$

$P_{12} = 1$ $I_{\gamma 1} \approx I_{\gamma 2} = 1$

$\varepsilon_{T1} \approx \varepsilon_{T2} = \varepsilon_T$



$$\varepsilon_T = \frac{N_T - A \cdot \varepsilon_T}{A \cdot (1 - \varepsilon_T)} = 1 - \sqrt{1 - \frac{N_T}{A}}$$

Total Efficiency calibration : remarks

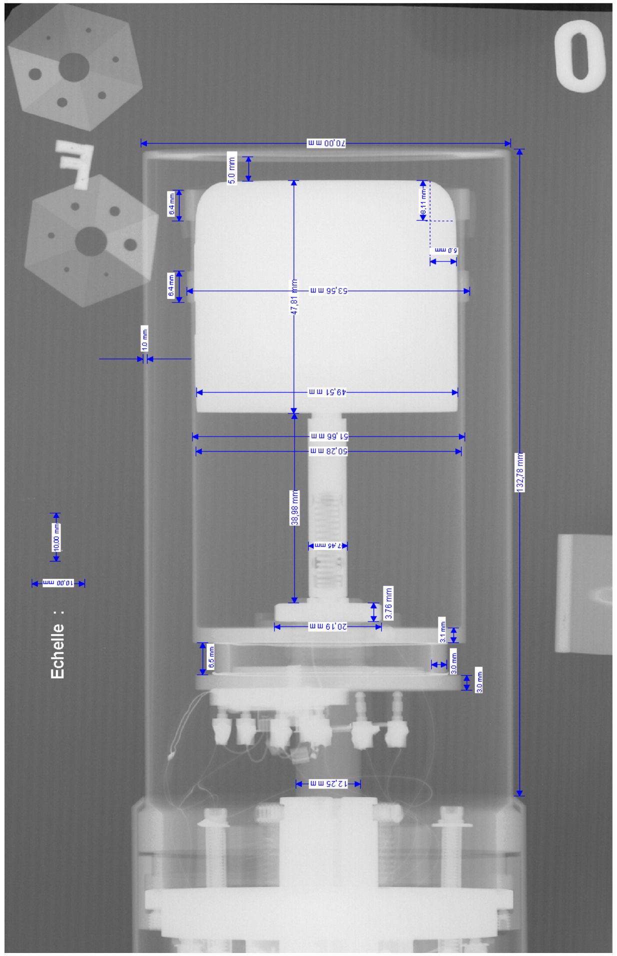
- Efficiency calibration for reference geometry
  - Difficult, even with point sources:
    - relative uncertainty 5-10 %
    - But limited influence on the coincidence correction factor

- Monte Carlo simulation ?

MONTE CARLO SIMULATION

Monte Carlo simulation

- Difficulties: bad knowledge of the detector internal parameters
- Accurate description is time-consuming:
  - Radiography (external dimensions)
  - Collimated beam (hole - dead layer)
  - Window to crystal distance (source at different distance)
  - Comparison with some experimental values
    - Different energies
    - Different geometries



Radiographie du détecteur G8 - Avril 2008 -

X-ray tube ->

Geometrical dimensions

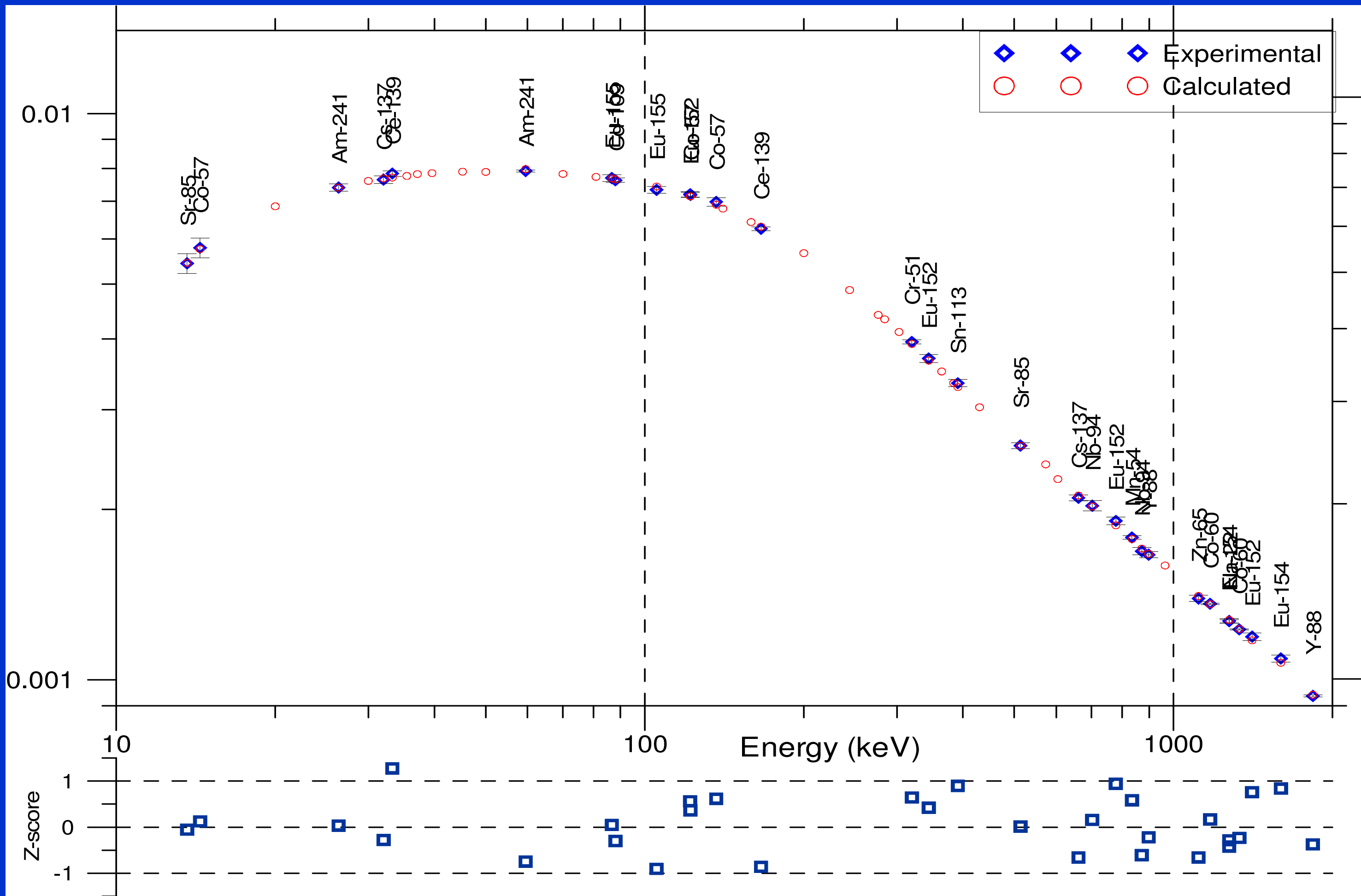
Cristal shape (rounding)

^{60}Co source ->

Hole dimensions

Comparison with experimental data

Extended-range coaxial HPGe with carbon-epoxy window (61 x 61 mm)
Point sources at 16 cm from the detector window



Picture from V. Peyres - CIEMAT

Monte Carlo simulation

- Can provide
  - FEP efficiency
  - Total efficiency (very dependent on the environment)
  - Absolute calculation should be compared with accurate experimental data

  - Strong interest for efficiency transfer calculation