Dataset and statistical analysis

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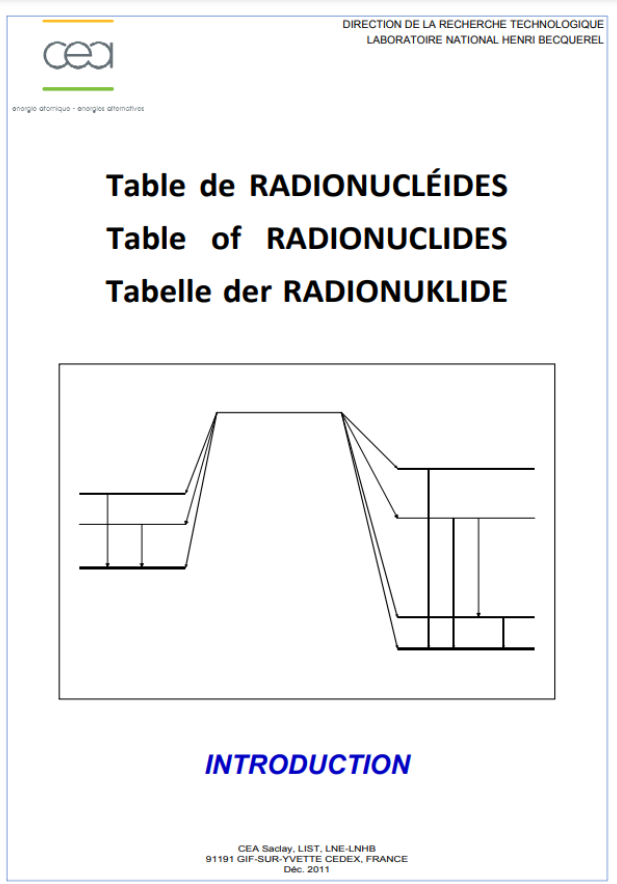
DDEP guidelines

The recommendations on data analysis are available on the chapter 6 of the Introduction to the Table of Radionuclides.

Chapter 6

RULES FOR EVALUATION AND COMPILATIONS



- The chapter provides advices and recommendations on
  - Rules for evaluation
  - Uncertainty evaluations
  - Data analysis

Rules for evaluations

6.1 Rules for evaluation

All intermediate stages in the compilation and evaluation of a decay parameter are not presented in detail in order to avoid unnecessary complexity. The main stages comprise the following:

- critical analysis of published results and, if necessary, correction of these results to account for more recent values hitherto unavailable to the original experimentalists; as a rule, results without associated uncertainties are discarded, and the rejection of values is documented;
- data obtained through private communications are only used when there is no published article available;
- adjustments may be made to the reported uncertainties;
- only one result, generally the latest one, is taken into account per laboratory (or author);
- recommended values are derived from an analysis of all available measurements (or theoretical considerations), along with the standard deviations corresponding to the 1 σ confidence level.

Making a recommendation: a two-steps process

The statistical analysis is performed at the very end of any evaluation and leads to a recommended value with an associated uncertainty. It is usually performed in two steps:

1. Refining the dataset
   - ❖ Unify the reported uncertainties
   - ❖ Identify the possible outliers
   - ❖ Remove any untrustworthy works

2. Making an average of the selected measurements
   1. Assert the dataset consistency
   2. Calculate a value and associated uncertainty using an averaging method

1. Refine the dataset

Are all uncertainties comparable?

Depending on the scientific community involved and on the period of publication, the methodology for uncertainty assessment and notations used are likely to differ.

- Some of the different notations that you might find
  - Error
  - Statistical and systematically errors
  - Statistical and systematically uncertainties
  - Type A and type B uncertainties
  - Non-linear combination of the above

- In order to perform the calculation it is necessary to unify the notation of all references
  - For all references, a single uncertainty is usually calculated

$$u = \sqrt{\sum {u_i}^2}$$

Chauvenet’s criterion

In order to help the evaluator to identify outliers from the dataset, the DDEP recommends the usage of the Chauvenet’s criterion

Statistical rejection of data

The rejection of data is based on the Chauvenet’s criterion [76Ch W, 72De D] that identifies statistical outliers.

In a set of n individual values a_i $(i = 1 \cdots n)$, having an arithmetical mean value of $\bar{a}$ and a standard deviation of σ, a measured value a_k is identified as outlier if:

$$a_k > \bar{a} + \sigma \times C \tag{6.15}$$

where:

C : depends on the number of measurements n and is calculated assuming that a_i $(i = 1 \cdots n)$ follows a Gaussian distribution.

a_k : is rejected if the probability to obtain such value is less than 50 %.

- This criterion is quite old (1863) and does not take into account measurement uncertainties
  - Other tests exist to identify outliers such as Peirce's criterion, Grubbs's test

Example of application to ^{137}Cs half-life

Reference	Author	Half-life estimation (year)	Associated uncertainty (year)
1955Br06	Brownn & Hall	29,94	0,35
1963Go03	Gorbics & Kunz	29,68	0,05
1973Di01	Dietz & Pachucki	30,174	0,011
1990Ma15	Martin & Taylor	30,029	0,012
2010Sc08	Schrader	30,057	0,041
2012Be08	Bellotti & Broggini	29,96	0,08
2016JU05	Juget & Nedjadi	29,994	0,029
2020UN0	Unterweger & Fitzgerlard	29,88	0,15

Full list					
	0,000913398	0,011569029			29,94
				1-OUT(CHV)	
				2-OUT(CHV)	
MIN	0,777022791	0,032286466			30,029
	0,066562333	0,128167369			30,057
	0,017483013	0,128715248			29,96
	0,133045519	0,216586363			29,994
	0,004972946	0,184777404			29,88
CONSISTENT DATA SET	Chi2 :	0,70210188	Chi2 crit.:	3,020481669	Nb Inp. Val.:
UWM :	29,97666667	uc(UWM) :	0,026114704		Nb. iter.:
WM :	30,02417856	uc(WM)int. :	0,010577868	uc(WM)ext. :	0,008863356
LWM :	30,02417856	uc(LWM) :	0,010577868	Min. Inp. Unc. :	0,012
LWM :	30,024	uc(LWM) :	0,011	(rounded values)	30,024 (11)
LWM has used weighted average and internal					
LWM has used weighted average and internal uncertainty					

Evaluator's choice

Based on the evaluator opinion, additional datum can be removed or some uncertainties can be increased **as long as the choice is tracked in the comment**

Reference	Author	Half-life estimation (year)	Associated uncertainty (year)
1990Ma15	Martin & Taylor	30,029	0,022
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Increased from 0.012 to 0.022 y

Without 1973Di01

MIN	0,500001063	0,056518311			30,029
	0,149871767	0,222426241			30,057
	0,039364756	0,132892551			29,96
	0,299565327	0,17592368			29,994
	0,011197086	0,212604262			29,88
CONSISTENT DATA SET	Chi2 :	0,800365045	Chi2 crit.:	3,323434591	Nb Inp. Val.:
UWM :	29,984	uc(UWM) :	0,030696905		Nb. iter.:
WM :	30,01832709	uc(WM)int. :	0,015872443	uc(WM)ext. :	0,014199983
LWM :	30,01832709	uc(LWM) :	0,015872443	Min. Inp. Unc. :	0,022447
LWM :	30,018	uc(LWM) :	0,016	(rounded values)	30,018 (16)
LWM has used weighted average and					
LWM has used weighted average and internal uncertainty					

2. Determine a recommended value

Basic considerations

Assuming a dataset of N measurements, each of them reporting a measured value m_i and associated uncertainty u_i

- One can calculate the weighted mean and associated *internal & external uncertainty*

$$\bar{M} = \frac{\sum {m_i}/{u_i}^2}{\sum 1/{u_i}^2} \qquad u_{int}(\bar{M}) = \sqrt{\frac{1}{\sum 1/{u_i}^2}} \qquad u_{ext}(\bar{M}) = \sqrt{\frac{\sum {(m_i - \bar{M})^2}/{u_i}^2}{(n-1)\sum 1/{u_i}^2}}$$

- The *consistency* of the dataset is estimated using the χ^2 ratio

$$\frac{\chi^2}{(n-1)} = \frac{u_{ext}(\bar{M})^2}{u_{int}(\bar{M})^2}$$

  - If $\chi^2 \sim 1$ then the uncertainties are consistent and $\bar{M}$ and $u_{int}(\bar{M})$ can be recommended
  - If $\chi^2 \gg 1$ then at least one of the uncertainty is underestimated

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  - If $\chi^2 \gg 1$ then at least one of the uncertainty is underestimated ⟵ **Troubles ahead**

Various approaches

For inconsistent dataset, a large number of approaches have been proposed over the years

- Unweighted mean
- Limitation of Relative Statistical Weights Method ⟵ DDEP recommendation
- Normalized Residuals Method
- Rajeval Technique
- Expected Value Method
- Mandel-Paule Method
- …

The choice is yours

Calculation tools

Different calculation codes are available and offer to evaluators the possibility to perform the statistical analysis.

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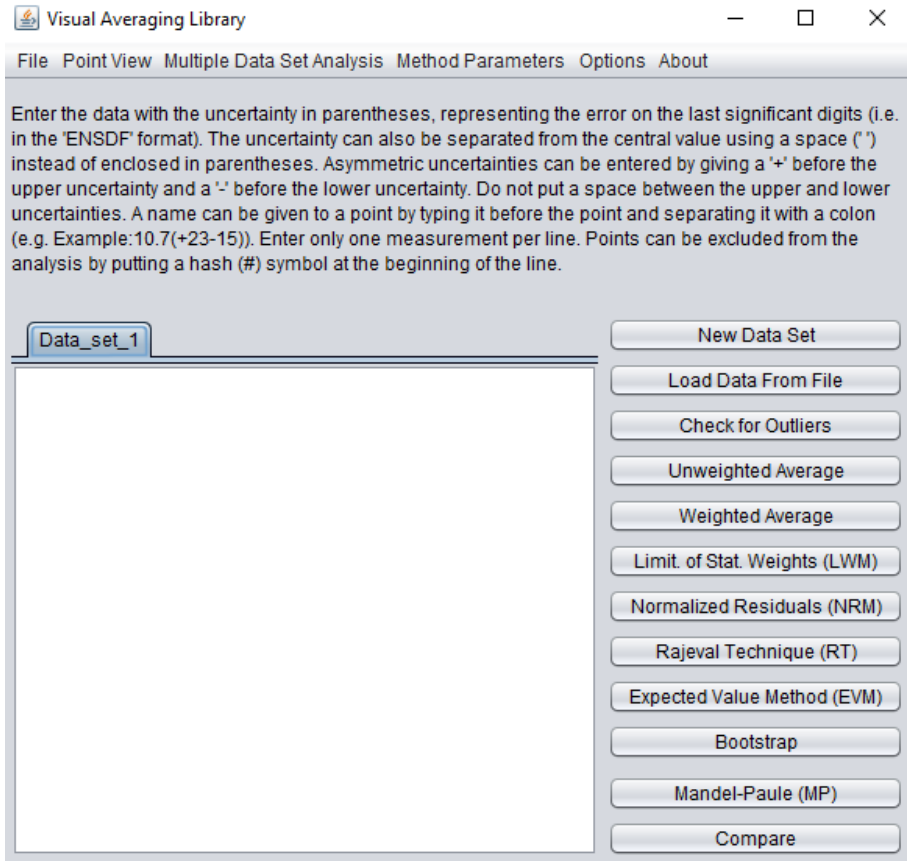
Lweight



Avelib

Some ideas to keep in mind

- Uncertainty propagation
  - Keep in mind that all the formula rely on the assumption $\frac{u(m)}{m} \ll 1$

- Several unwritten rules exist within the community. It's up to you to decide what to do with them.
  - Example: a recommended uncertainty cannot be smaller than the smallest of the reported experimental uncertainties.

CONSISTENT DATA SET	Chi2 :	0,800365045	Chi2 crit.:	3,323434591	Nb Inp. Val.:
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LNE – LNHB/CEA **Table de Radionucléides** $^{137}_{55}\mathrm{Cs}_{82}$

$T_{1/2}(^{137}\mathrm{Cs}) \quad : \quad 30{,}018 \quad (22) \quad \mathrm{a}$

3. Conclusions

Conclusion

- Due to the large discrepancy between the various communities when it comes to uncertainty evaluation, the averaging of different experimental measurements can be difficult and tedious.

- DDEP provides guidelines and calculation tools to help you to use an approach as rigorous as possible

- However, each evaluation is unique and often requires the evaluator to **make personal decision** which might differs from the usual practices.
  - These choices can have sometimes big impact on recommended values and associated uncertainties
  - They need to be tracked down in the comment file