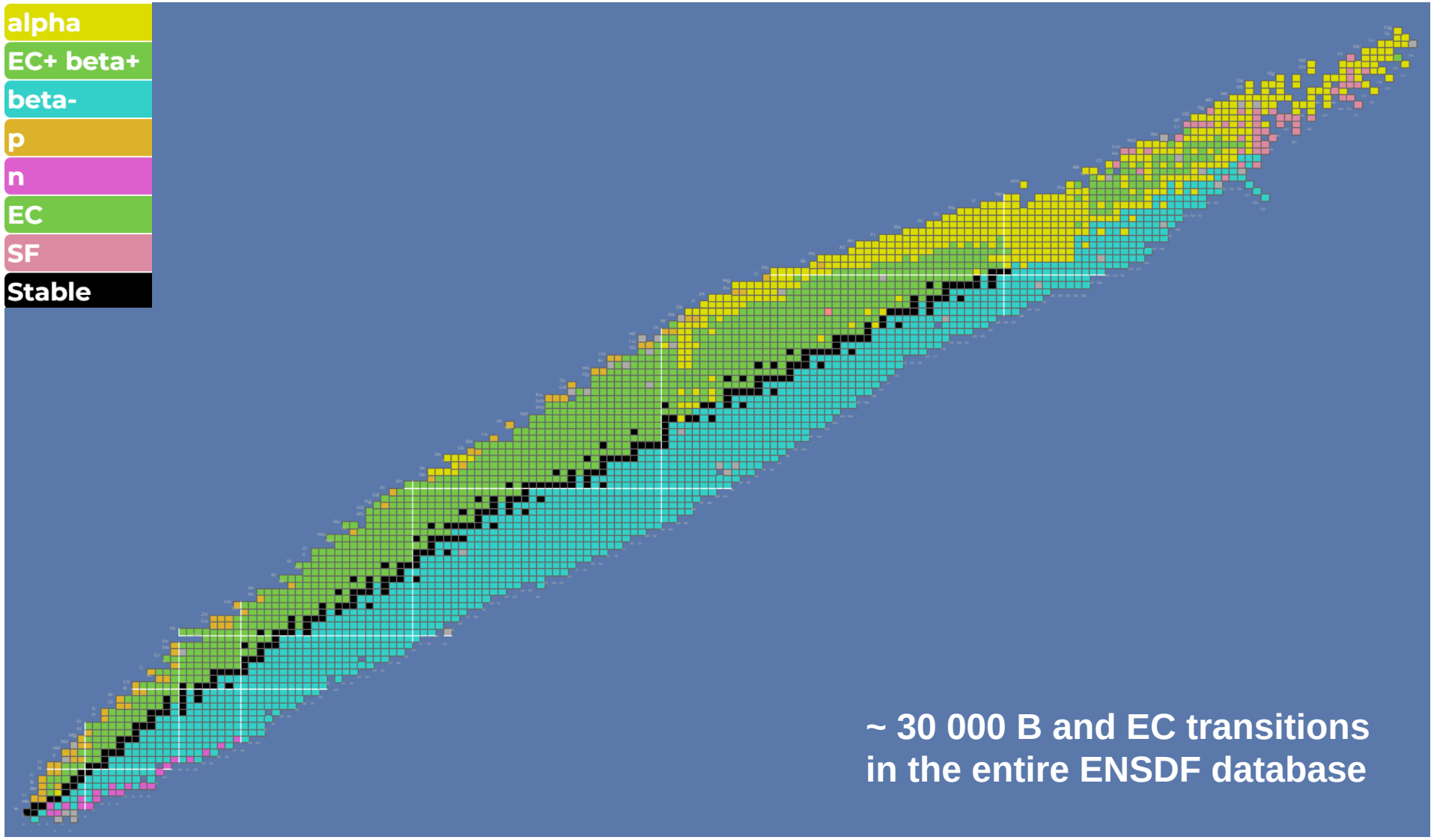
Beta decays and electron captures

X. Mougeot, CEA-LNHB (France)

DDEP Training Workshop

21-25 October 2024, CEA Saclay

The main radioactive decay mode



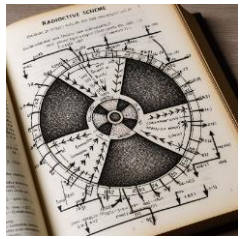
Importance of beta decays

Metrology

Activity measurements
(Liquid scintillation,
Ionization chambers)

Better knowledge → **Improvement of accuracy and uncertainties**

Nuclear decay data



ENSDF, **DDEP**, **JEFF** databases

Properties calculated with the LogFT legacy code
(Gove and Martin, 1971)

log-*ft* for Jπ assignment, **no beta spectrum**

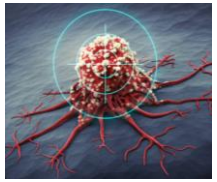
Fundamental research

Nuclear astrophysics (*r*-process)

Neutrino physics (reactor anomaly, reactor monitoring, non-proliferation)

Standard Model (CKM matrix unitarity, weak magnetism)

New physics (Fierz interference, sterile neutrino, dark matter)

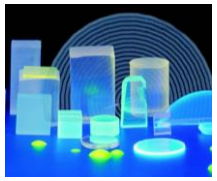


Medical applications

Micro-dosimetry, internal radiotherapy, contamination

Nuclear fuel cycle

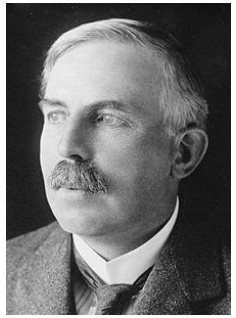
Decay heat, nuclear waste

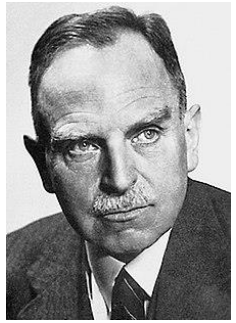


Developments

Beta-voltaic batteries, new detectors (e.g. $LaBr_3$)

Some historical aspects (1)





1898: E. Rutherford distinguishes α and β rays.

1910: O. Hahn and L. Meitner build the first β spectrometer and observe some rays.

→ Two-body disintegration is claimed. Unknown radiochemical substances are identified.

1914: J. Chadwick uses proportional counter from H. Geiger and observes continuous spectra.

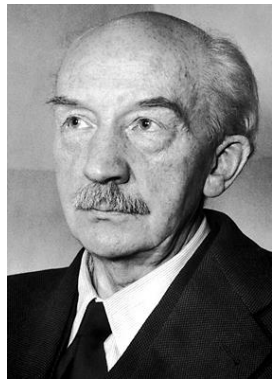
→ With a two-body disintegration, energy is not conserved.

1922-1923: Discovery of Compton effect. For L. Meitner, continuity of beta spectra is due to a secondary phenomenon.

1927-1929: With a fine calorimetry measurement, C. Ellis demonstrates that beta spectra are continuous.

→ End of 15 years controversy, but still a mystery!

Some historical aspects (2)



1924: For H. Kramers, J. Slater and N. Bohr, energy conservation could be only statistical.

1925: First coincidence measurement from H. Geiger et W. Bothe.

→ Energy is conserved event-by-event in beta decay.

1930: W. Pauli invents the neutrino to save energy conservation.

"Today, I did what a theorist should never do in his life. I have indeed tried to explain something we cannot understand by something we cannot observe".

1933: E. Fermi establishes his theory of beta decay. Electron and neutrino are created at the moment of the decay. His quantitative calculations reproduce very well Sargent's rule (empirical relationship between E_{max} and $T_{1/2}$).

→ Article rejected by Nature editors: his theory was considered as "speculations too far from reality"!

Fermi theory

β^-:	$n \longrightarrow p + e^- + \bar{\nu}_e$	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+1}\mathrm{Y} + e^- + \bar{\nu}_e$	$E_{\max} = Q + E_{\mathrm{nuc}}^{\mathrm{initial}} - E_{\mathrm{nuc}}^{\mathrm{final}}$
β^+:	$p \longrightarrow n + e^+ + \nu_e$	${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + e^+ + \nu_e$	$E_{\max} = Q + E_{\mathrm{nuc}}^{\mathrm{initial}} - E_{\mathrm{nuc}}^{\mathrm{final}} - 2m_e$
Electron capture:	$p + e^- \longrightarrow n + \nu_e$	${}^{A}_{Z}\mathrm{X} + e^- \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + \nu_e$	$E_{\max} = Q + E_{\mathrm{nuc}}^{\mathrm{initial}} - E_{\mathrm{nuc}}^{\mathrm{final}}$

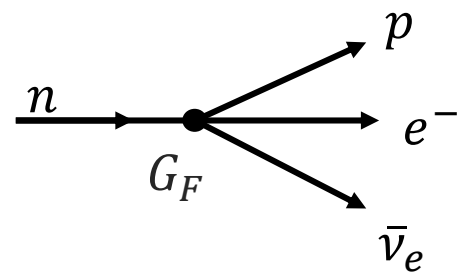
Fermi theory is a low-energy effective theory

→ Weak interaction mediated by $W^{\pm}$ bosons.

→ However, $m_W \approx 80\ \mathrm{GeV} \gg E_{\max}(\mathrm{decay}) \leq 30\ \mathrm{MeV}$.

→ Instantaneous contact interaction of 4 fermions.

→ Fermi coupling constant: $G_F/(\hbar c)^3 = 1.1163787(6) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$.



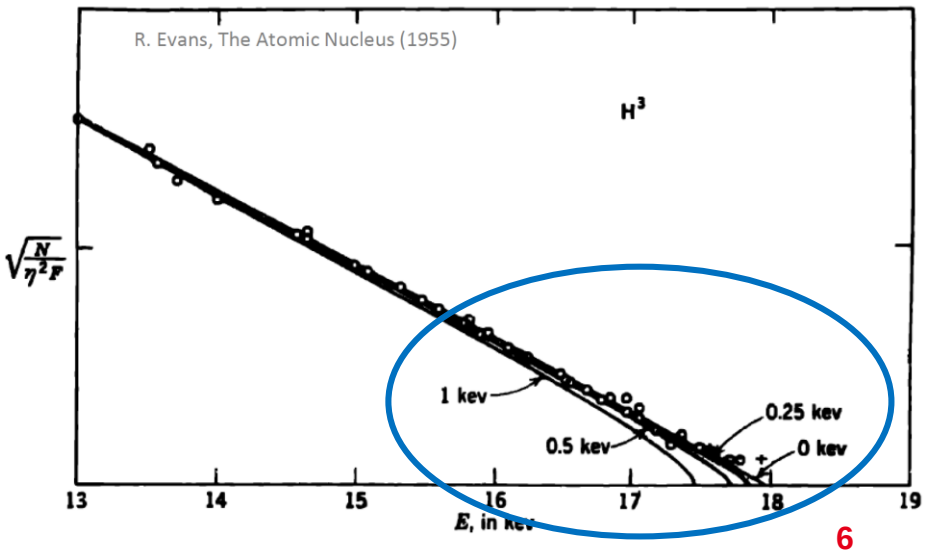
It is common to assume a **massless neutrino** and an **infinitely massive nucleus** (max. recoil energy in ^{7}Be decay: 57 eV).

Fermi's golden rule: weak perturbation → transition probability

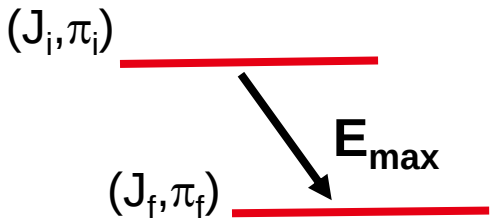
$$\Gamma = \frac{2\pi}{\hbar} |\langle \Psi_{\mathrm{final}} | H' | \Psi_{\mathrm{initial}} \rangle|^2 \rho(E_{\mathrm{final}})$$

matrix element

density of final states



Selection rules



$$J_i = J_f + L_\beta + S_\beta \qquad \Delta J = L_\beta + S_\beta \qquad \pi_i\pi_f = (-1)^{L_\beta}$$

Type of transition	Order of forbiddenness (L_β)	ΔJ	$\pi_i\pi_f$
Allowed		0, ±1	+1
Forbidden unique (Independent of nuclear structure in first, but good approximation)	1st	±2	–1
	2nd	±3	+1
	$(\Delta J-1)^{th}$	>1	$(-1)^{(\Delta J-1)}$
Forbidden non-unique (Sensitive to nuclear structure →)	1st	0, ±1	–1
	2nd	±2	+1
	3rd	±3	–1
	$(\Delta J)^{th}$	>1	$(-1)^{(\Delta J)}$

Comparative half-lives

Beta decay half-lives span several order of magnitudes. Direct comparison is not practical.

Partial half-life $t = T_{1/2}/P_{\text{trans}}$

Comparative half-lives ft defined by

$$f = \int_1^{W_0} pWq^2F(Z,W)\mathrm{d}W$$

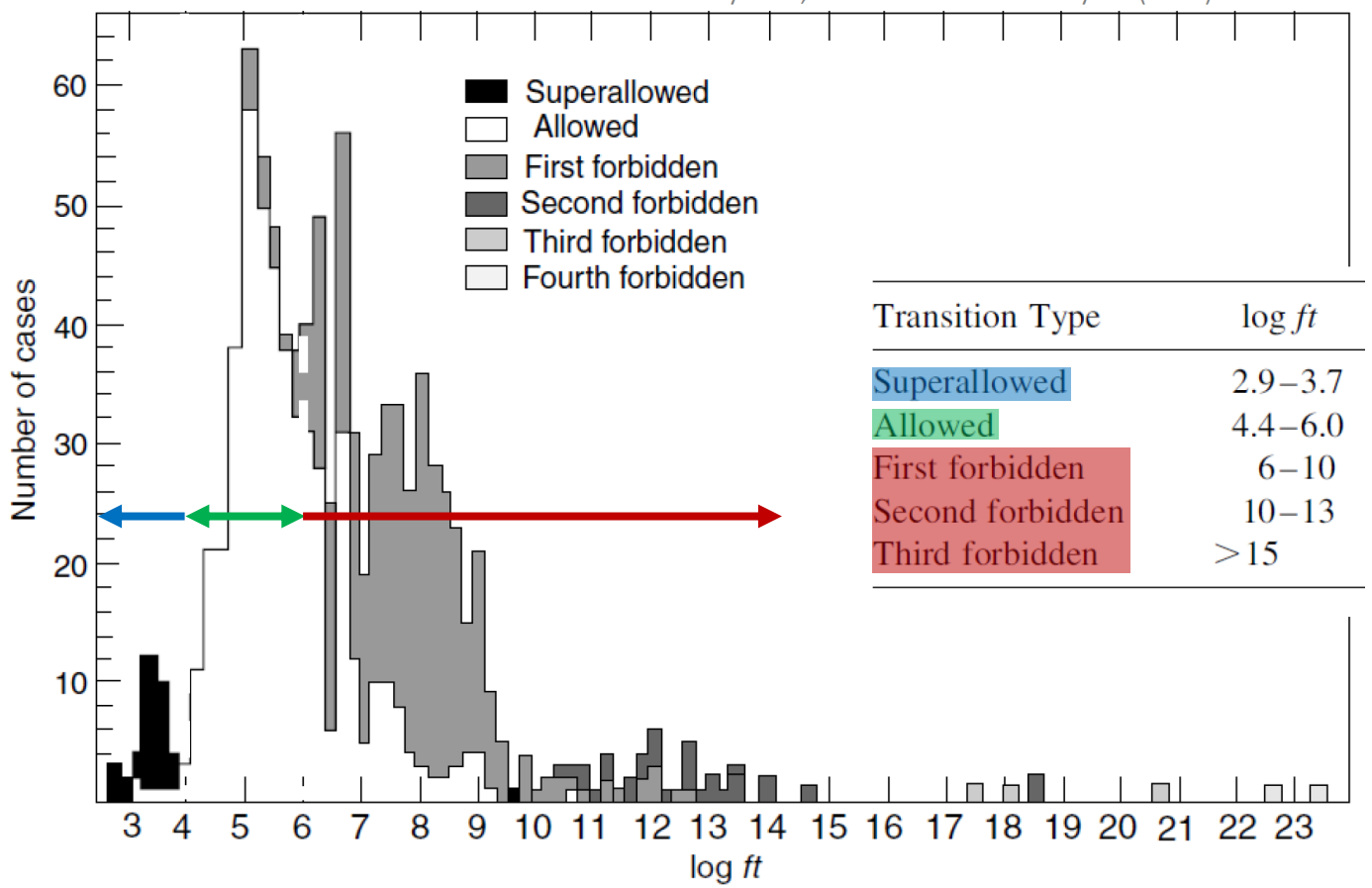
$$ft = \frac{2\pi^3 \ln 2}{G_F^2 \overline{C(W)}}$$

(in relativistic units, $\hbar = m_e = c = 1$)

$\log_{10} ft$ values are usually compared.

N.B. Gove, M.J. Martin, *Log-f tables for beta decay*, Nuclear Data Tables 10, 205-317 (1971).

W. Meyerhof, Elements of Nuclear Physics (1967)



Transition Type	log *ft*
Superallowed	2.9–3.7
Allowed	4.4–6.0
First forbidden	6–10
Second forbidden	10–13
Third forbidden	>15

Behrens and Bühring formalism

Similarly we obtain for the space components

$$\langle p|\,\mathbf{V}+\mathbf{A}\,|n\rangle = iu_p^+\underbrace{\gamma_4\gamma_\mu}(1+\underbrace{\lambda\gamma_5})u_n = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p}{2W_p}}$$

$$\begin{pmatrix}0 & i\boldsymbol{\sigma}\\ i\boldsymbol{\sigma} & 0\end{pmatrix}\lambda\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$\times\left\{\left(\frac{\boldsymbol{\sigma}\mathbf{p}}{W_p+M_p}\chi_p^{m'}\right)^+\boldsymbol{\sigma}\chi_n^m+(\chi_p^{m'})^+\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}}{W_n+M_n}\chi_n^m-\lambda(\chi_p^{m'})^+\boldsymbol{\sigma}\chi_n^m\right.$$

$$\left.-\lambda\left[\left(\frac{\boldsymbol{\sigma}\mathbf{p}}{W_p+M_p}\chi_p^{m'}\right)^+\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}}{W_n+M_n}\chi_n^m\right]\right\}. \quad (6.38)$$

This equals to

$$\langle p|\,\mathbf{V}+\mathbf{A}\,|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}\left\{(\chi_p^{m'})^+\underbrace{\frac{\boldsymbol{\sigma}\mathbf{p}_p}{W_p+M_p}}_{\frac{\mathbf{p}_p+i(\boldsymbol{\sigma}\times\mathbf{p}_p)}{W_p+M_p}}\boldsymbol{\sigma}\chi_n^m\right.$$

$$+(\chi_p^{m'})^+\underbrace{\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}_n}{W_n+M_n}}_{\frac{\mathbf{p}_n-i(\boldsymbol{\sigma}\times\mathbf{p}_n)}{W_n+M_n}}\chi_n^m-\lambda(\chi_p^{m'})^+\boldsymbol{\sigma}\chi_n^m$$

$$\left.-\lambda\left[(\chi_p^{m'})^+\underbrace{\frac{\boldsymbol{\sigma}\mathbf{p}_p}{W_p+M_p}\boldsymbol{\sigma}\frac{\boldsymbol{\sigma}\mathbf{p}_n}{W_n+M_n}}\chi_n^m\right]\right\}$$

$$\frac{-(\mathbf{p}_p\mathbf{p}_n)\boldsymbol{\sigma}+(\boldsymbol{\sigma}\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\boldsymbol{\sigma}\mathbf{p}_n)-i(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}. \quad (6.39)$$

Finally we obtain for the space components

$$\langle p|\,\mathbf{V}(0)+\mathbf{A}(0)\,|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}$$

$$\times\left\{\left[\frac{\mathbf{p}_p}{W_p+M_p}+\frac{\mathbf{p}_n}{W_n+M_n}\right](\chi_p^{m'})^+\chi_n^m+(\chi_p^{m'})^+\right.$$

$$\times\left[\frac{i(\boldsymbol{\sigma}\times\mathbf{p}_p)}{W_p+M_p}-\frac{i(\boldsymbol{\sigma}\times\mathbf{p}_n)}{W_n+M_n}\right]\chi_p^m-\lambda(\chi_p^{m'})^+\boldsymbol{\sigma}\chi_n^m$$

$$+\lambda\frac{\mathbf{p}_p\mathbf{p}_n}{(W_p+M_p)(W_n+M_n)}\{(\chi_p^{m'})^+\boldsymbol{\sigma}\chi_n^m\}+\lambda\frac{i(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}$$

$$\left.\times(\chi_p^{m'})^+\chi_n^m-\lambda\left[(\chi_p^{m'})^+\frac{(\boldsymbol{\sigma}\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\boldsymbol{\sigma}\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}\chi_n^m\right]\right\}. \quad (6.40)$$

$$-\frac{i}{2M_A}F_M(q^2)(\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma}-F_S(q^2)q_0+\frac{1}{4(2M_A)^2}F_S(q^2)q_0(\mathbf{P}^2-\mathbf{q}^2)$$

$$\left.-\frac{i}{2}\frac{1}{(2M_A)^2}F_S(q^2)q_0(\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma}\right\}\chi^{M_i} \quad (9.15)$$

$$\langle\phi_f(p_f)|\,A_0(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\left\{-\frac{1}{2M_A}F_A(q^2)(\mathbf{P}\boldsymbol{\sigma})\right.$$

$$-\frac{q_0}{2M_A}F_P(q^2)(\mathbf{q}\boldsymbol{\sigma})-F_T(q^2)(\mathbf{q}\boldsymbol{\sigma})+\frac{1}{4}\frac{1}{(2M_A)^2}F_T(q^2)$$

$$\left.\times[(\mathbf{P}\mathbf{q})(\boldsymbol{\sigma}\mathbf{P}+\boldsymbol{\sigma}\mathbf{q})-(\boldsymbol{\sigma}\mathbf{q})(\mathbf{P}^2-\mathbf{q}^2)]\right\}\chi^{M_i} \quad (9.16)$$

$$\langle\phi_f(p_f)|\,\mathbf{V}(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\left\{\frac{1}{2M_A}F_V(q^2)\mathbf{P}+\frac{i}{2M_A}F_V(q^2)(\boldsymbol{\sigma}\times\mathbf{q})\right.$$

$$+iF_M(q^2)(\boldsymbol{\sigma}\times\mathbf{q})-\frac{1}{2M_A}F_M(q^2)q_0\mathbf{q}-\frac{i}{4M_A}F_M(q^2)q_0(\boldsymbol{\sigma}\times\mathbf{P})$$

$$-F_S(q^2)\mathbf{q}+\frac{1}{4(2M_A)^2}F_S(q^2)\mathbf{q}(\mathbf{P}^2-\mathbf{q}^2)-\frac{i}{2(2M_A)^2}F_S(q^2)\mathbf{q}$$

$$\times((\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma})-\frac{i}{2(2M_A)^2}F_M(q^2)\mathbf{P}((\mathbf{P}\times\mathbf{q})\boldsymbol{\sigma})-\frac{i}{4(2M_A)^2}$$

$$\left.\times F_M(q^2)(\mathbf{P}^2+\mathbf{q}^2)(\boldsymbol{\sigma}\times\mathbf{q})+\frac{i}{2(2M_A)^2}F_M(q^2)(\mathbf{P}\mathbf{q})(\boldsymbol{\sigma}\times\mathbf{P})\right\}\chi^{M_i} \quad (9.17)$$

$$\langle\phi_f(p_f)|\,\mathbf{A}(0)\,|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\left\{-F_A(q^2)\boldsymbol{\sigma}+\frac{1}{2(2M_A)^2}\right.$$

$$\times F_A(q^2)\mathbf{P}^2\boldsymbol{\sigma}-\frac{1}{4(2M_A)^2}F_A(q^2)(\mathbf{P}^2+\mathbf{q}^2)\boldsymbol{\sigma}-\frac{i}{2(2M_A)^2}$$

$$\times F_A(q^2)(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_A(q^2)[(\boldsymbol{\sigma}\mathbf{P})\mathbf{P}-(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}]$$

$$+\frac{1}{2M_A}F_T(q^2)[(\mathbf{P}\mathbf{p})\boldsymbol{\sigma}-\mathbf{q}(\boldsymbol{\sigma}\mathbf{P})]-F_T(q^2)q_0\boldsymbol{\sigma}$$

$$+\frac{1}{2(2M_A)^2}F_T(q^2)q_0\mathbf{P}^2\boldsymbol{\sigma}-\frac{1}{4(2M_A)^2}F_T(q^2)q_0(\mathbf{P}^2+\mathbf{q}^2)\boldsymbol{\sigma}$$

$$-\frac{i}{2(2M_A)^2}F_T(q^2)q_0(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_T(q^2)q_0[(\boldsymbol{\sigma}\mathbf{P})\mathbf{P}$$

$$\left.-(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}]-\frac{1}{2M_A}F_P(q^2)(\boldsymbol{\sigma}\mathbf{q})\mathbf{q}\right\}\chi^{M_i}. \quad (9.18)$$

$${}^AF_{KK0}(\mathbf{q}^2) = \lambda\,{}^A\mathfrak{M}_{KK0}(\mathbf{q}^2)+\frac{f_T}{R}\left[\sqrt{\{K(2K+1)\}}\,{}^D\mathfrak{N}_{KK-11}(\mathbf{q}^2)\right.$$

$$\left.-\sqrt{\left\{\frac{K+1}{(2K+3)^2(2K+1)}\right\}}(qR)^2\,{}^D\mathfrak{N}_{KK+11}(\mathbf{q}^2)\right]$$

$$-\frac{f_P}{R}\{W_0R+\tfrac{6}{5}\alpha Z\}\,{}^D\mathfrak{N}_{KK0}(\mathbf{q}^2) \quad (8.263b)$$

$${}^VF_{KK1}(\mathbf{q}^2) = {}^V\mathfrak{M}_{KK1}(\mathbf{q}^2)+\frac{f_M}{R}\left[\sqrt{\{(K+1)(2K+1)\}}\,{}^D\mathfrak{N}_{KK-11}(\mathbf{q}^2)\right.$$

$$+\sqrt{\left\{\frac{K}{(2K+3)^2(2K+1)}\right\}}(qR)^2\,{}^D\mathfrak{N}_{KK+11}(\mathbf{q}^2)$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^C\mathfrak{N}_{KK1}(\mathbf{q}^2)\right] \quad (8.263c)$$

$${}^AF_{KK1}(\mathbf{q}^2) = \lambda\,{}^A\mathfrak{M}_{KK1}(\mathbf{q}^2)+\frac{f_T}{R}\left[\sqrt{\{(K+1)(2K+1)\}}\,{}^C\mathfrak{N}_{KK-11}(\mathbf{q}^2)\right.$$

$$+\sqrt{\left\{\frac{K}{(2K+3)^2(2K+1)}\right\}}(qR)^2\,{}^C\mathfrak{N}_{KK+11}(\mathbf{q}^2)$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^D\mathfrak{N}_{KK1}(\mathbf{q}^2)\right] \quad (8.263d)$$

$$-{}^VF_{KK-11}(\mathbf{q}^2) = {}^V\mathfrak{M}_{KK-11}(\mathbf{q}^2)+\frac{f_M}{R}\left[\sqrt{\left\{\frac{K+1}{(2K+1)^3}\right\}}(qR)^2\,{}^D\mathfrak{N}_{KK1}(\mathbf{q}^2)\right.$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^C\mathfrak{N}_{KK-11}(\mathbf{q}^2)\right]-\frac{f_S}{R}\sqrt{\left\{\frac{K}{(2K+1)^3}\right\}}$$

$$\times(qR)^2\,{}^C\mathfrak{N}_{KK0}(\mathbf{q}^2) \quad (8.263e)$$

$$-{}^AF_{KK-11}(\mathbf{q}^2) = \lambda\,{}^A\mathfrak{M}_{KK-11}(\mathbf{q}^2)+\frac{f_T}{R}\left[\sqrt{\left\{\frac{K+1}{(2K+1)^3}\right\}}(qR)^2\,{}^C\mathfrak{N}_{KK1}(\mathbf{q}^2)\right.$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^D\mathfrak{N}_{KK-11}(\mathbf{q}^2)\right]-\frac{f_P}{R}\sqrt{\left\{\frac{K}{(2K+1)^3}\right\}}$$

$$\times(qR)^2\,{}^D\mathfrak{N}_{KK0}(\mathbf{q}^2) \quad (8.263f)$$

$$-{}^VF_{KK+11}(\mathbf{q}^2) = {}^V\mathfrak{M}_{KK+11}(\mathbf{q}^2)+\frac{f_M}{R}\left[\sqrt{\left\{\frac{K(2K+3)^2}{2K+1}\right\}}\,{}^D\mathfrak{N}_{KK1}(\mathbf{q}^2)\right.$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^C\mathfrak{N}_{KK+11}(\mathbf{q}^2)\right]$$

$$+\frac{f_S}{R}\sqrt{\left\{\frac{(K+1)(2K+3)^2}{2K+1}\right\}}\,{}^C\mathfrak{N}_{KK0}(\mathbf{q}^2) \quad (8.263g)$$

$$-{}^AF_{KK+11}(\mathbf{q}^2) = \lambda\,{}^A\mathfrak{M}_{KK+11}(\mathbf{q}^2)+\frac{f_T}{R}\left[\sqrt{\left\{\frac{K(2K+3)^2}{2K+1}\right\}}\,{}^C\mathfrak{N}_{KK1}(\mathbf{q}^2)\right.$$

$$\left.+(W_0R+\tfrac{6}{5}\alpha Z)\,{}^D\mathfrak{N}_{KK+11}(\mathbf{q}^2)\right]$$

$$+\frac{f_P}{R}\sqrt{\left\{\frac{(K+1)(2K+3)^2}{2K+1}\right\}}\,{}^D\mathfrak{N}_{KK0}(\mathbf{q}^2). \quad (8.263h)$$

$$\left.+\sqrt{\tfrac{2}{3}}\left(\int rI'(r)\beta\gamma_5T_{121}\right)\right\}$$

$$\mp\frac{f_P}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^D\mathfrak{N}^{(0)}_{110}(1,1,1,1) \quad (14.101)$$

$${}^AF^{(0)}_{121} = \mp\lambda\,{}^A\mathfrak{M}^{(0)}_{121}-\frac{f_T}{R}\left[\frac{5}{\sqrt{3}}\,{}^C\mathfrak{N}^{(0)}_{111}-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{121}\right]\mp\frac{f_P}{R}5\sqrt{\tfrac{2}{3}}\,{}^D\mathfrak{N}^{(0)}_{110} \quad (14.102)$$

$${}^AF^{(0)}_{121}(1,1,1,1) = \mp\lambda\,{}^A\mathfrak{M}^{(0)}_{121}(1,1,1,1)$$

$$-\frac{f_T}{R}\left\{\sqrt{\tfrac{2}{3}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{111}\right)\right.$$

$$\left.-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{121}(1,1,1,1)\right\}$$

$$\mp\frac{f_P}{R}\sqrt{\tfrac{2}{3}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta\gamma_5T_{110}\right) \quad (14.103)$$

$${}^VF^{(0)}_{211} = -{}^V\mathfrak{M}^{(0)}_{211}-\frac{f_M}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^C\mathfrak{N}^{(0)}_{211} \quad (14.104)$$

$${}^VF^{(0)}_{220} = {}^V\mathfrak{M}^{(0)}_{220}+\frac{f_M}{R}\sqrt{(10)}\,{}^C\mathfrak{N}^{(0)}_{211}\pm\frac{f_S}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^V\mathfrak{M}^{(0)}_{220} \quad (14.105)$$

$${}^VF^{(0)}_{220}(1,1,1,1) = {}^V\mathfrak{M}^{(0)}_{220}(1,1,1,1)$$

$$+\frac{f_M}{R}\left\{\sqrt{\tfrac{2}{3}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{211}\right)\right.$$

$$\left.+\sqrt{\tfrac{3}{5}}\left(\int\left(\frac{r}{R}\right)rI'(r)\beta T_{231}\right)\right\}$$

$$\pm\frac{f_S}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^V\mathfrak{M}^{(0)}_{220}(1,1,1,1) \quad (14.106)$$

$${}^AF^{(0)}_{221} = \pm\lambda\,{}^A\mathfrak{M}^{(0)}_{221}+\frac{f_T}{R}[\sqrt{(15)}\,{}^C\mathfrak{N}^{(0)}_{211}-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{221}] \quad (14.107)$$

$${}^AF^{(0)}_{221}(1,1,1,1) = \pm\lambda\,{}^A\mathfrak{M}^{(0)}_{221}(1,1,1,1)$$

$$+\frac{f_T}{R}\left\{\sqrt{\tfrac{3}{5}}\left(\int\left(\frac{r}{R}\right)[5I(r)+rI'(r)]\beta T_{211}\right)\right.$$

$$-\sqrt{\tfrac{2}{5}}\left(\int\left(\frac{r}{R}\right)rI'(r)\beta T_{231}\right)$$

$$\left.-(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{221}(1,1,1,1)\right\} \quad (14.108)$$

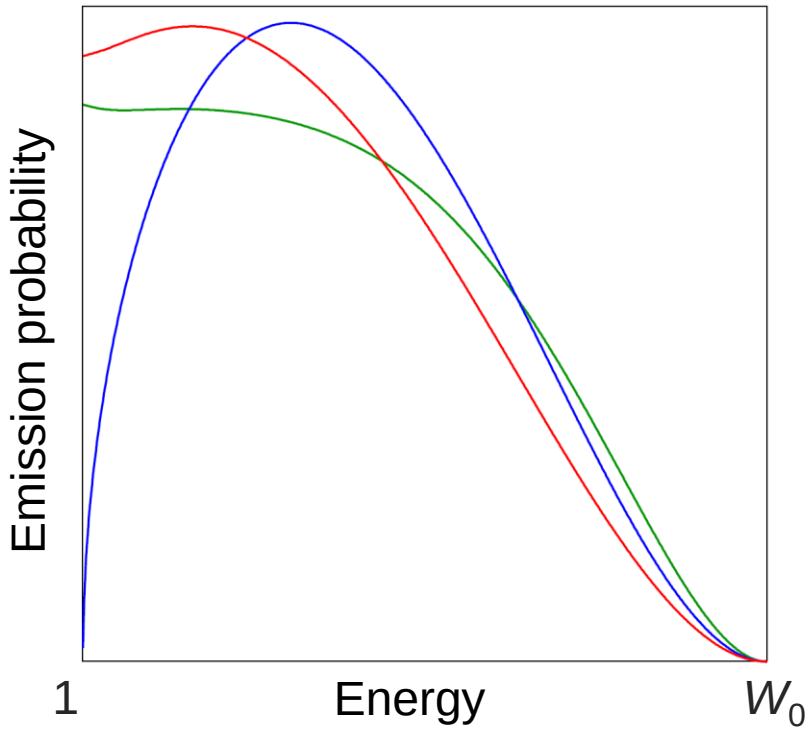
H. Behrens and W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

More than 600 p.!

Beta spectrum shape

Phase space Fermi function Shape factor

$$\frac{\mathrm{d}P}{\mathrm{d}W} \propto \boxed{pWq^2} \cdot \boxed{F(Z,W)} \cdot \boxed{C(W)}$$

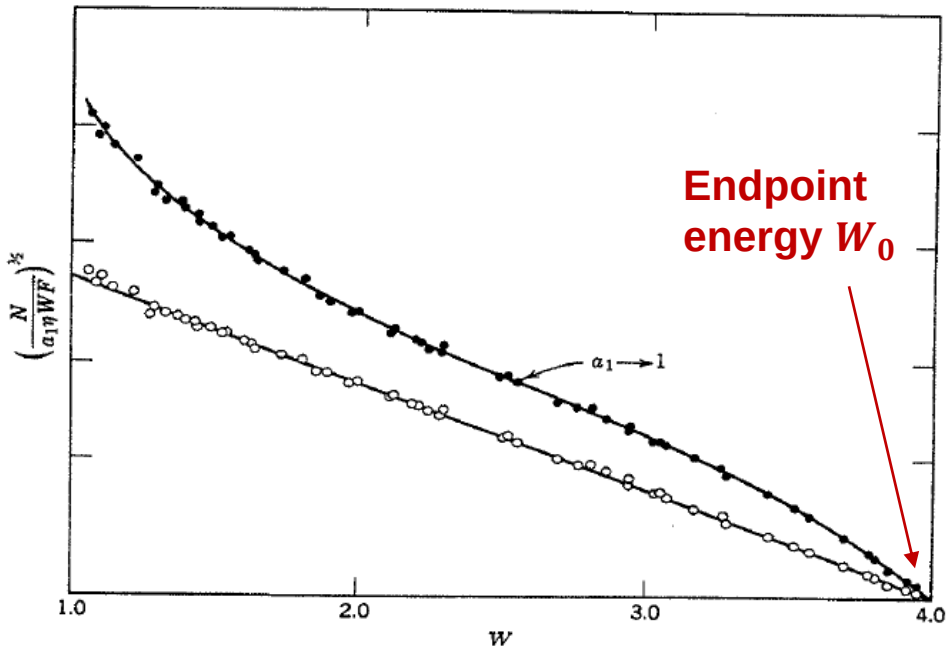


$W = 1 + E_{\mathrm{kin}}/m_e$ electron energy

$W_0 = 1 + E_{\mathrm{max}}/m_e$ transition energy

Kurie plot: $\sqrt{N_{\mathrm{exp}}/pWF(Z,W)} = (W_0 - W)$

- Straight line for allowed transitions.
- Shape factor needed otherwise, in fit or Kurie plot.



$p = \sqrt{W^2 - 1}$ electron momentum

$q = W_0 - W$ neutrino momentum

Shape factors

Allowed: $C(W) = 1$

First forbidden unique: $C(W) = q^2 + \lambda_2 p^2$

Second forbidden unique: $C(W) = q^4 + \lambda_2 q^2 p^2 + \lambda_3 p^4$

Third forbidden unique: $C(W) = q^6 + \lambda_2 q^4 p^2 + \lambda_3 q^2 p^4 + \lambda_4 p^6$

Etc.

Much more complicated for forbidden non-unique transitions. Nuclear and lepton currents must be folded.

In Behrens and Bühring formalism:

$$F(Z,W) = \frac{\alpha_{-1}^2 + \alpha_{+1}^2}{2p^2} \qquad \lambda_k = \frac{\alpha_{-k}^2 + \alpha_{+k}^2}{\alpha_{-1}^2 + \alpha_{+1}^2}$$

Coulomb amplitudes of the relativistic electron wave functions, determined by normalization conditions.

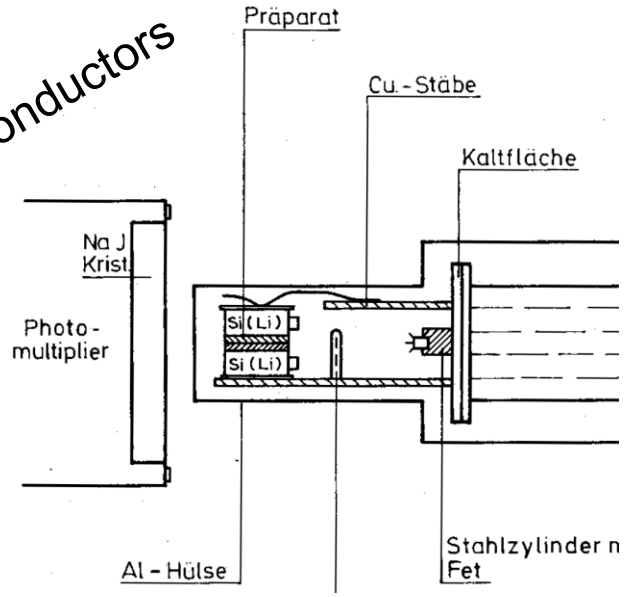
Tabulated unscreened parameters and screened-to-unscreened ratios can be found in

Behrens, Jänecke, Landolt-Börnstein, New Series, Group I, vol. 4, Springer Verlag, Berlin (1969)

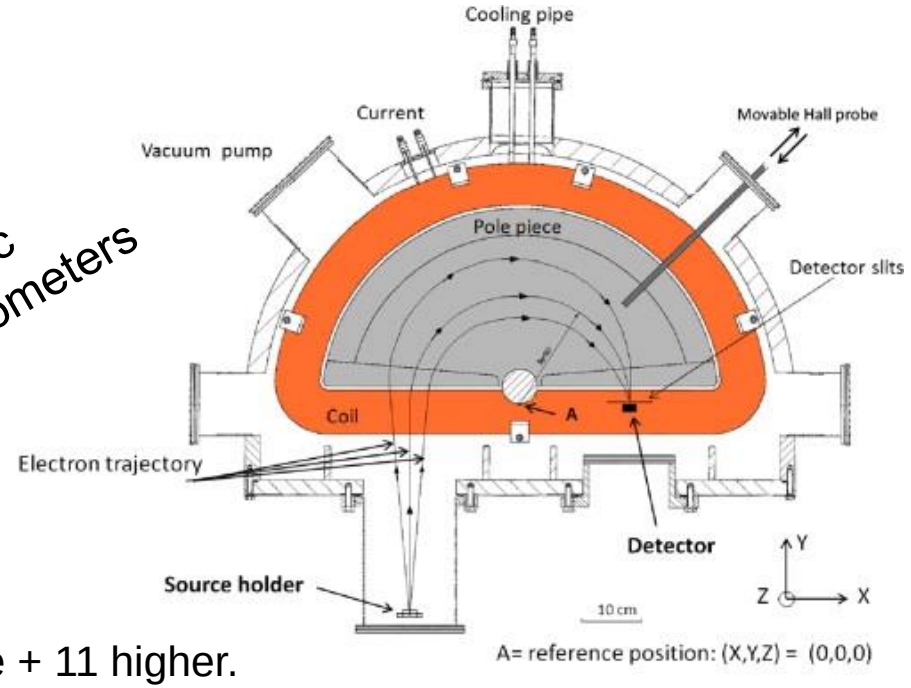
Reviews of measured beta spectra

- 1976 review: H. Behrens, L. Szybisz, Physik Daten Vol. 6-1, Zent. Atomker.-Dok. (1976).
- 2015 update + detailed calculations of beta and neutrino spectra: X. Mougeot, Phys. Rev. C 91, 055504 (2015).
- Created database of 130 experimental shape factors, essentially from

semiconductors



magnetic spectrometers



- Transitions: 36 allowed, 25 1st unique + 5 higher, 53 1st non-unique + 11 higher.

→ Very few (6) measurements below 20 keV.

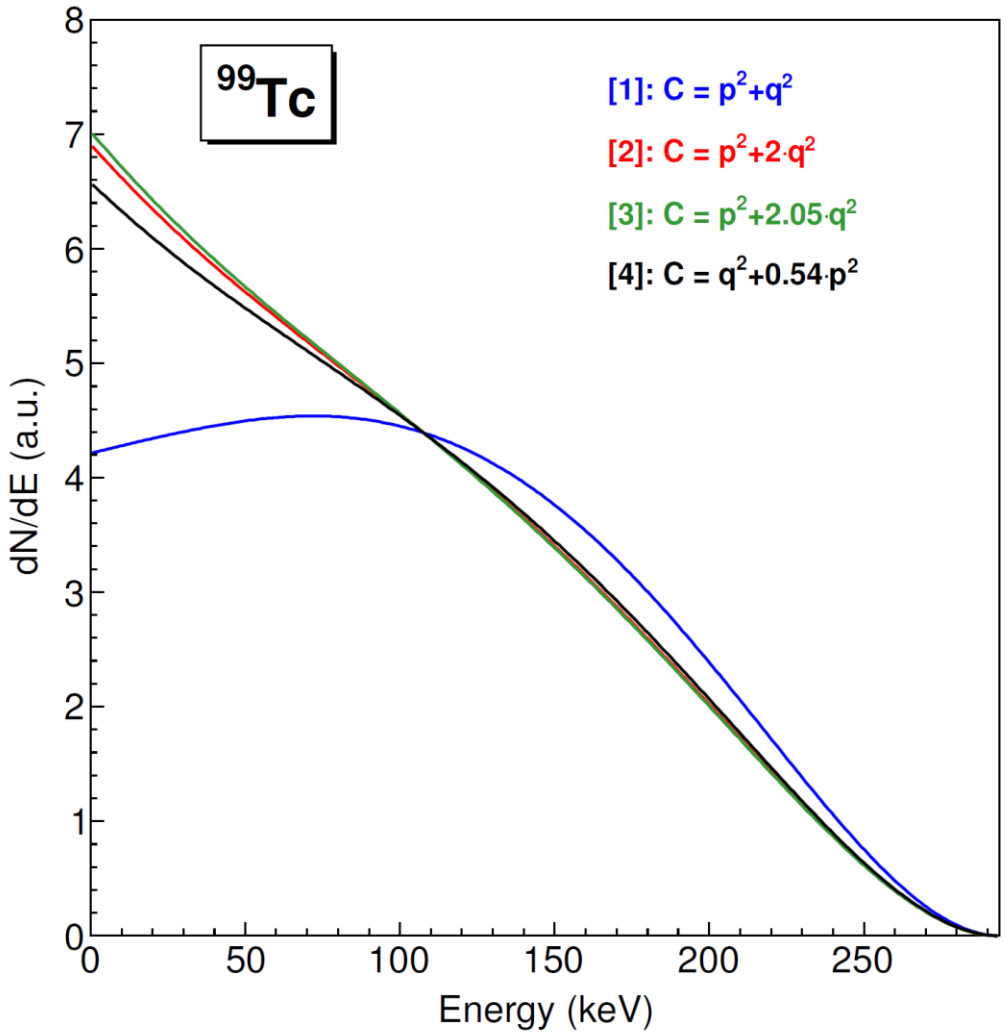
→ Very few high order forbidden transitions.

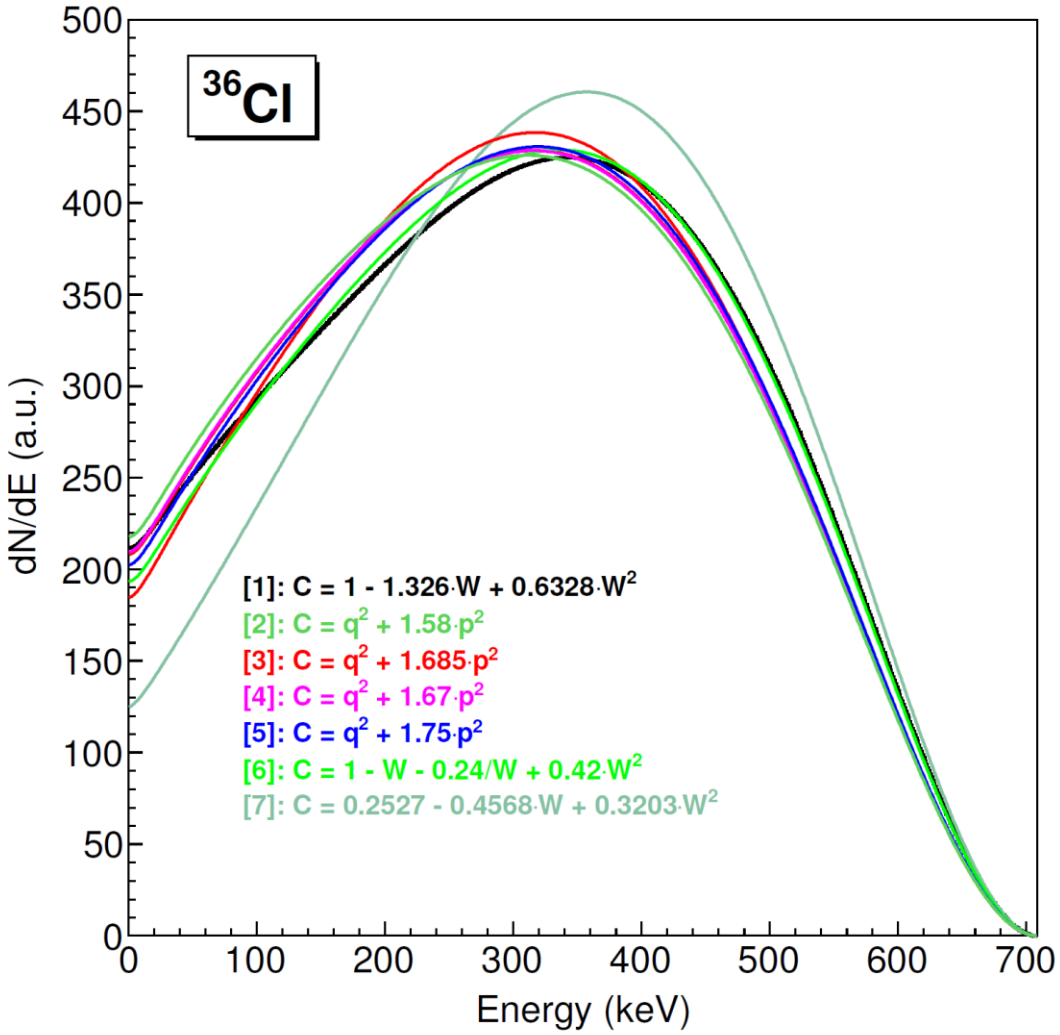
→ 11 published shape factors since 1976!

New precise measurements are needed to test the theoretical predictions.

Inconsistent measurements

Comparison of published experimental spectra for two second forbidden non-unique decays.

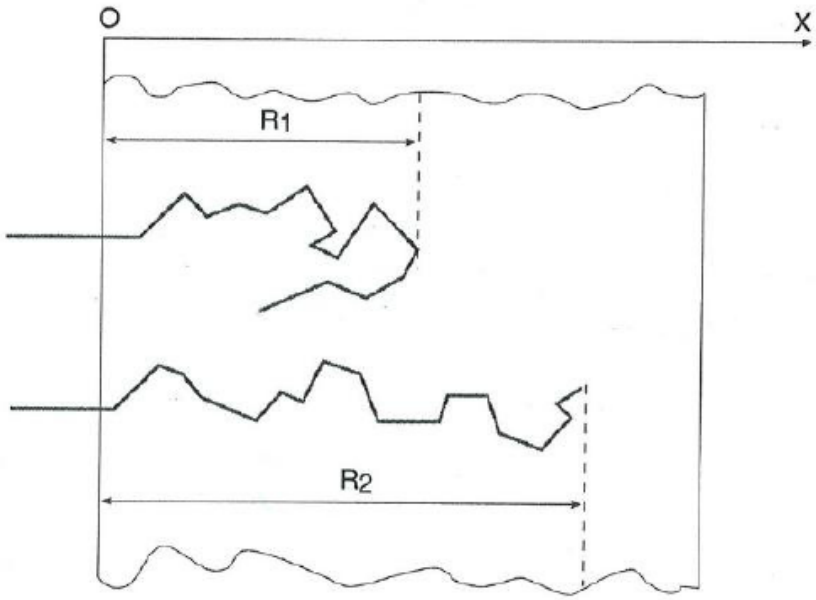




Interaction with matter

Electrons scatter very easily in matter:

- Chaotic path, high angular straggling.
- Elastic collisions.
- Cherenkov effect.
- Inelastic collisions, excitation and ionisation of material atoms.
- Bremsstrahlung in material.



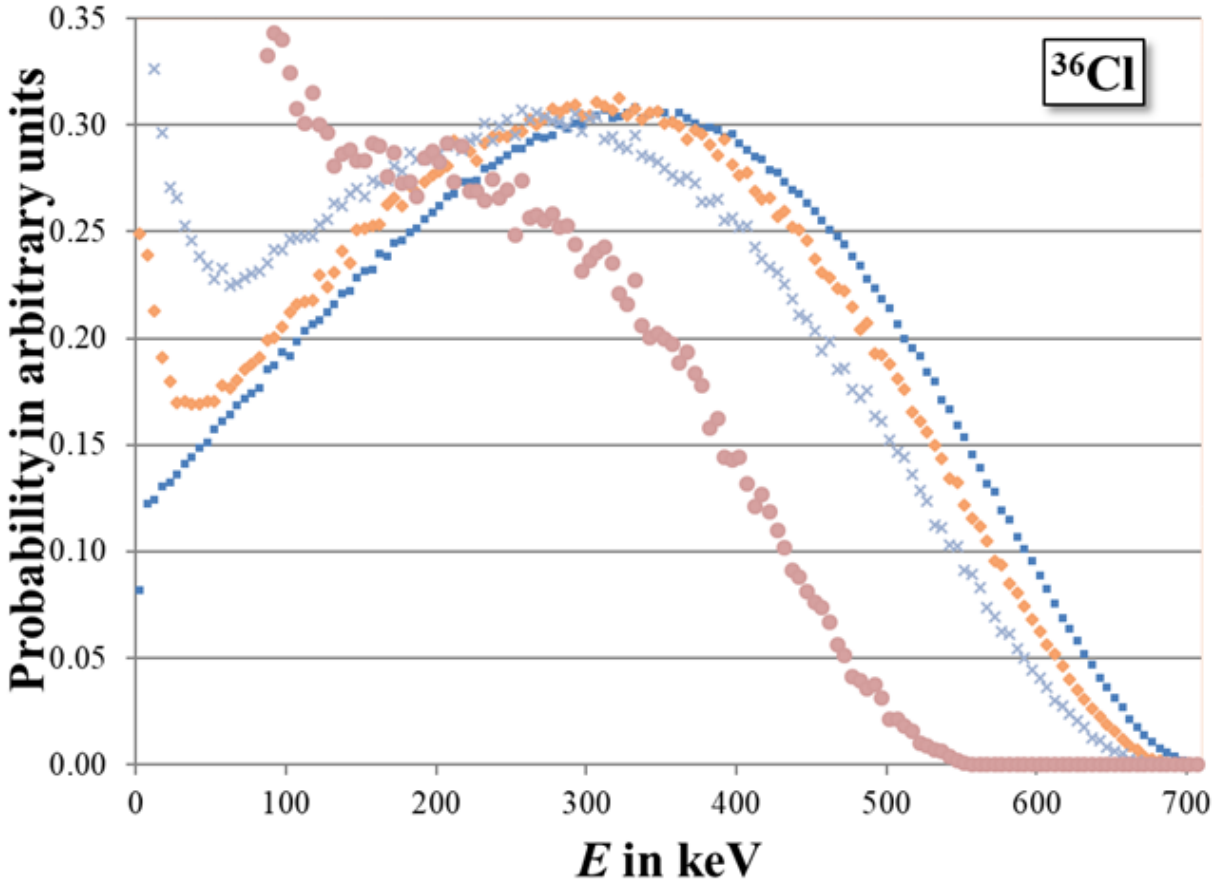
Some formulas to estimate electron range:

$$R\ (\text{in cm}) \approx \frac{0.35 \times E_{\text{MeV}}}{\varrho\ (\text{in g} \cdot \text{cm}^{-3})}$$

$$R(\text{in mg} \cdot \text{cm}^{-2}) \approx 412 \times E_{\text{MeV}}^{(1.265-0.0964\times \ln E)} \quad \text{from 10 keV to 2.5 MeV.}$$

$$R(\text{in mg} \cdot \text{cm}^{-2}) \approx (530 \times E_{\text{MeV}}) - 106 \quad \text{beyond 2.5 MeV.}$$

Self-absorption effect



- Self-absorption effect study for a ^{36}Cl radioactive source.
- NaCl crystals as sphere of various sizes.
- Monte Carlo simulation of deposited energy within a 4π absorber.

- **MC input spectrum from** → Rotzinger *et al.*, J. Low Temp. Phys. 151, 1087-1093 (2008)
- **MC, NaCl sphere, r = 0.05 mm**
- **MC, NaCl sphere, r = 0.1 mm**
- **MC, NaCl sphere, r = 0.5 mm**

Unpublished study performed by M. Paulsen, K. Kossert and J. Beyer (PTB Germany) within the **MetroBeta** project.

The BetaShape code

Current situation in ENSDF database

Weak interaction properties in nuclear decay data are incomplete and come from calculation: mean energies, capture probabilities, log-*ft* values.

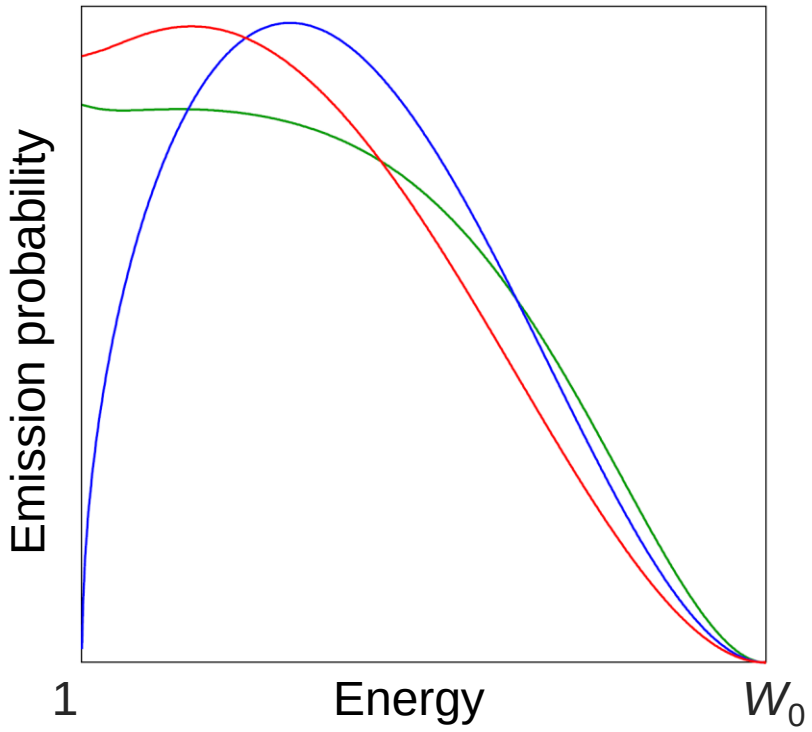
They have been determined for the last 50 years with the LogFT code, developed in the late 1960's.

Quite simple analytical models of beta transitions and electron captures. Limited in forbiddenness degree.

Beta spectrum shape

Phase space Fermi function Shape factor

$$\frac{\mathrm{d}P}{\mathrm{d}W} \propto \boxed{pWq^2} \cdot \boxed{F(Z,W)} \cdot \boxed{C(W)}$$



W electron energy, W_0 transition energy

p electron momentum, q neutrino momentum

Allowed	$C(W) = 1$
First forbidden unique	$C(W) = q^2 + \lambda_2 p^2$
Second forbidden unique	$C(W) = q^4 + \lambda_2 q^2 p^2 + \lambda_3 p^4$
Third forbidden unique	$C(W) = q^6 + \lambda_2 q^4 p^2 + \lambda_3 q^2 p^4 + \lambda_4 p^6$
	Etc.

- ✓ The BetaShape program (version 2.4) now replaces the LogFT code for the new ENSDF evaluations. Electron captures also treated.

  → Available on IAEA-NSDD GitHub: https://github.com/IAEA-NSDDNetwork

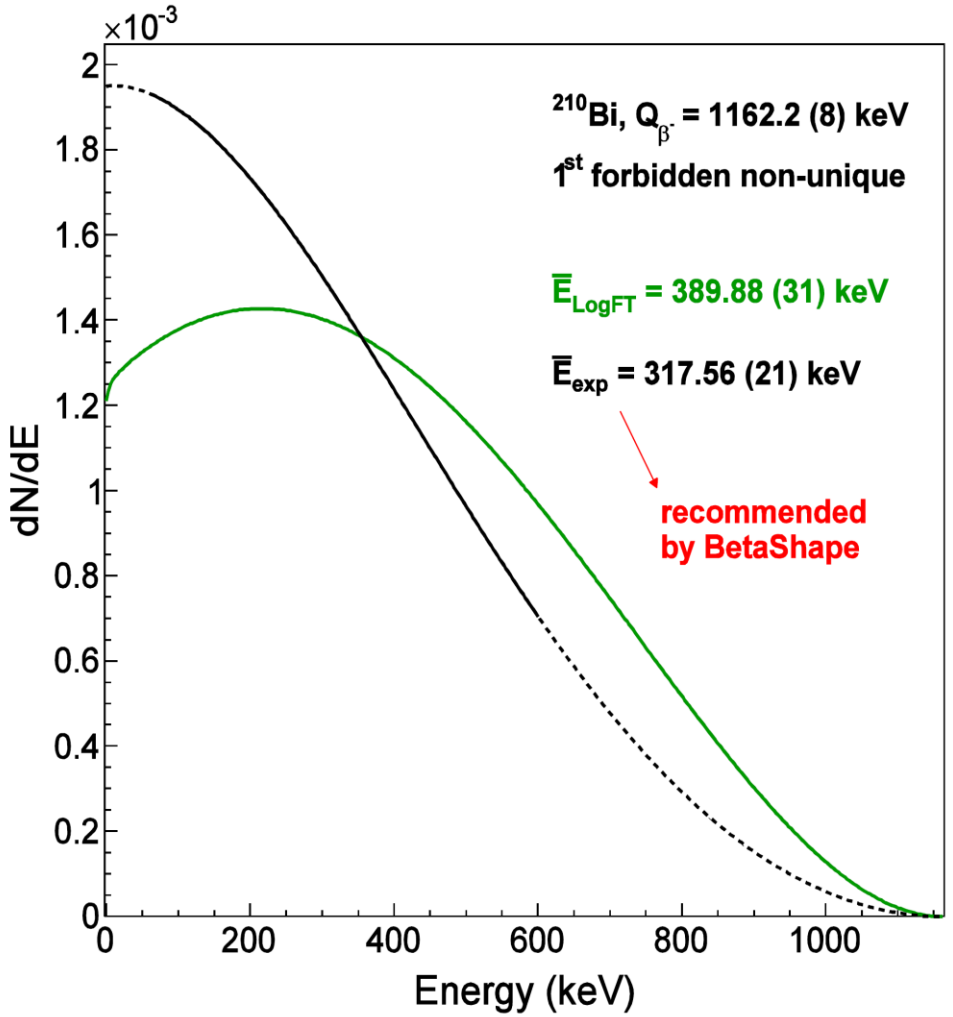
- ✓ $F(Z,W)$ and λ_k parameters determined from the relativistic electron wave functions, obtained by numerical solving of the Dirac equation.

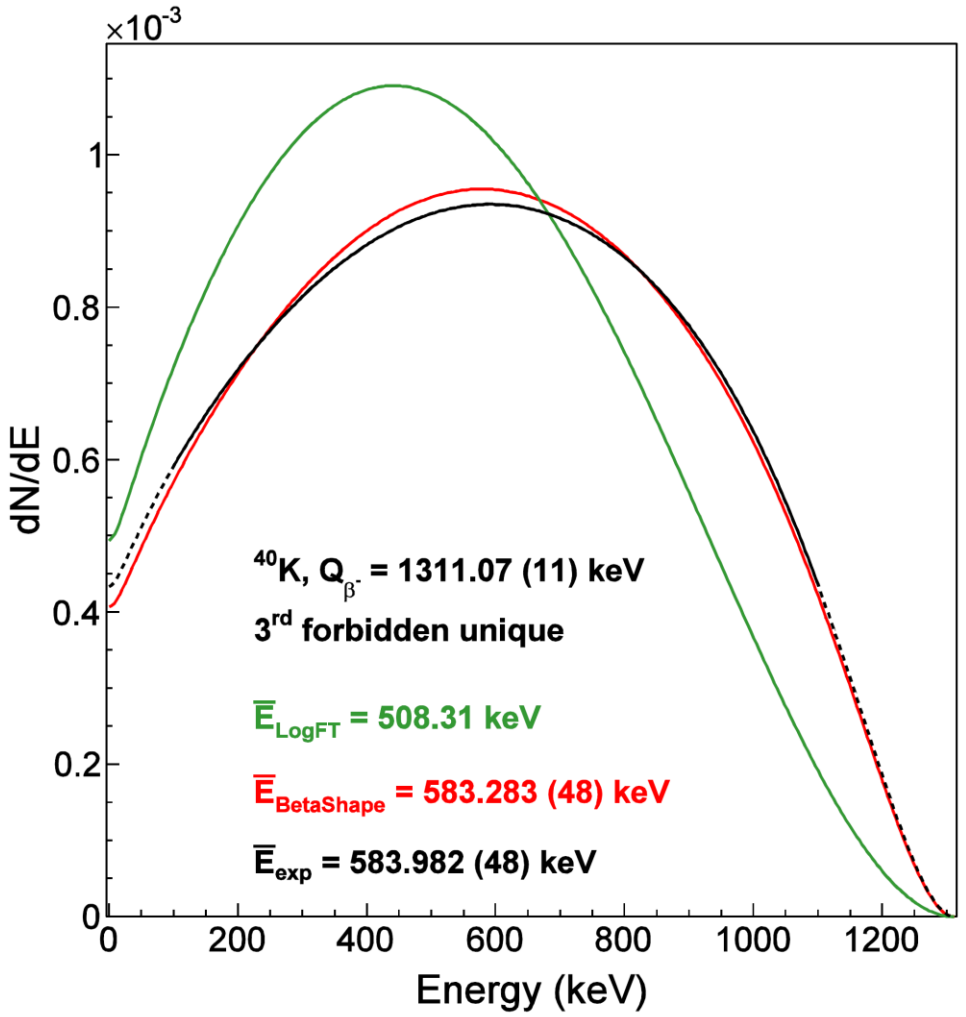
- ✓ Included: extended nucleus; atomic exchange, overlap and screening; radiative corrections; database of experimental shape factors.

For forbidden non-unique transitions, coupling with nuclear structure is necessary.

→ ξ-approximation possible but accuracy is questionable.

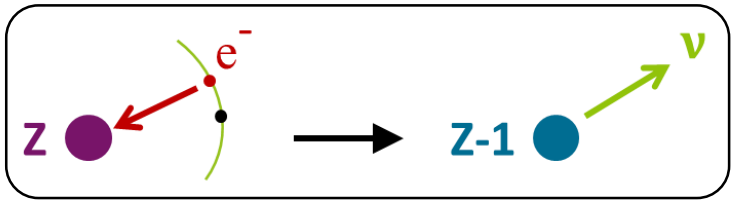
Examples of improved calculations





These two transitions are calculated as allowed by the LogFT program, which does not provide any beta spectrum.

Electron capture modelling



Total capture probability

$$\lambda_\varepsilon \propto \sum_{\kappa_x} n_{\kappa_x} C_{\kappa_x} q_{\kappa_x}^2 \beta_{\kappa_x}^2 B_{\kappa_x} \left(1 + \sum_{m,\kappa} P_{m\kappa}\right)$$

- n_{κ_x}: relative occupation number
- q_{κ_x}: ν momentum
- κ_x: shell quantum number
- C_{κ_x}: "shape" factor similar to $C(W)$ in β decay
- β_{κ_x}: amplitude of wave function
- B_{κ_x}: overlap and exchange corrections
- $\left(1 + \sum_{m,\kappa} P_{m\kappa}\right)$: shaking effects

+ hole effect (vacancy)
+ radiative corrections

Allowed and forbidden unique
→ no nuclear structure

If transition energy $\geq 2m_e$
→ **competition with a β^+ transition**

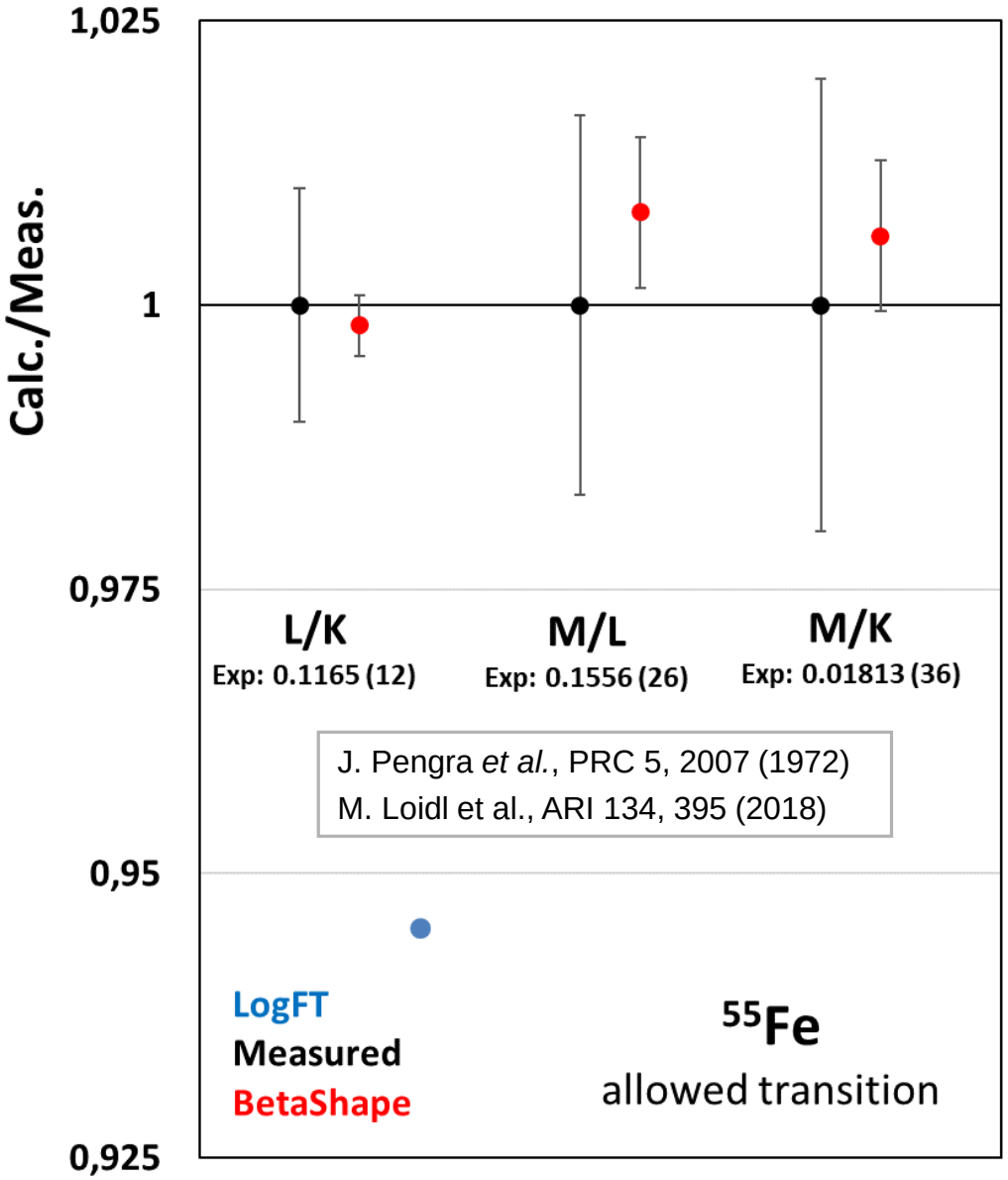
X. Mougeot, Appl. Radiat. Isot. 154, 108884 (2019)

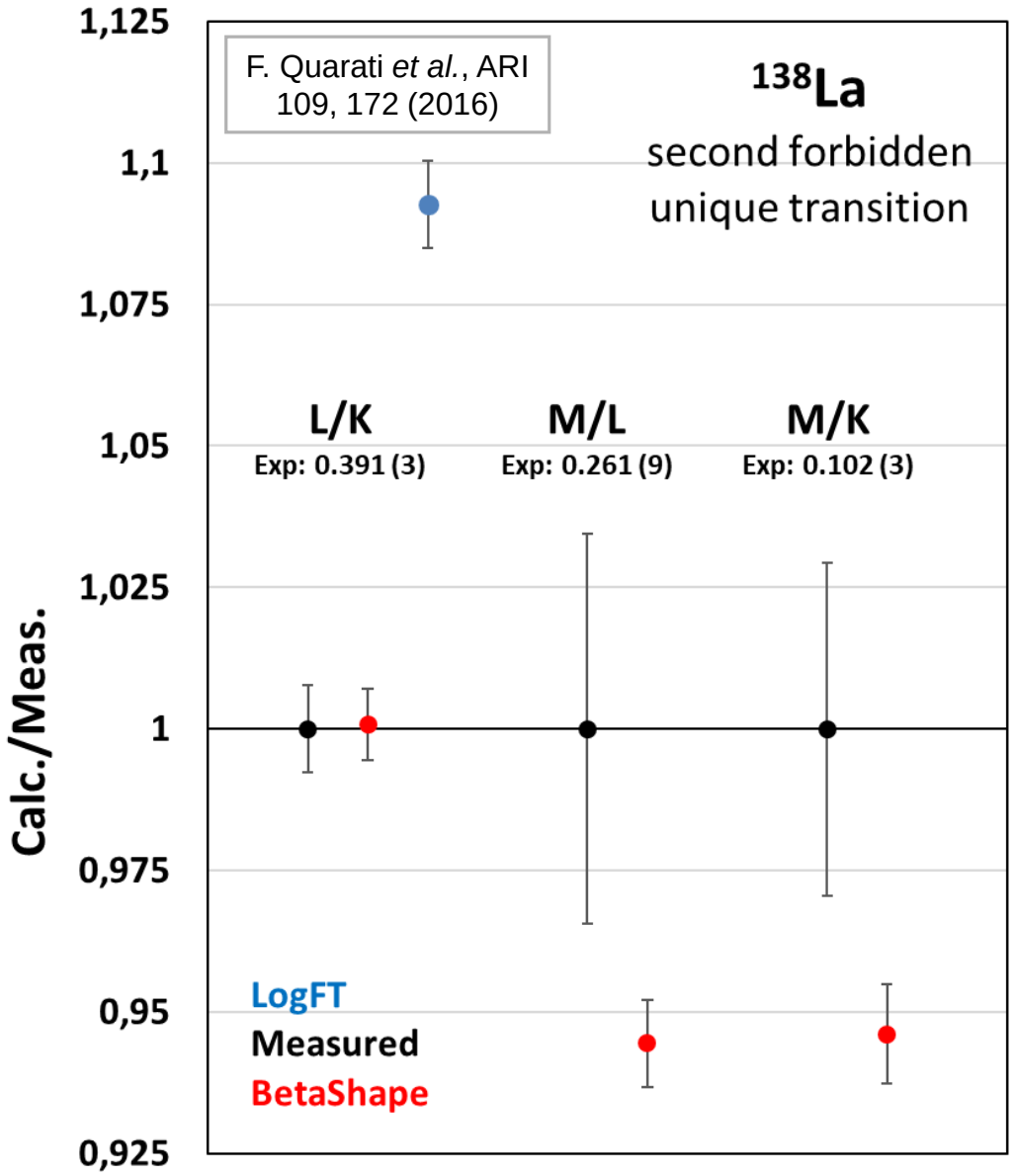
Atomic wave functions

- Numerical solving of Dirac equation
- Forced convergence to relativistic DFT energies, with electron correlations

S. Kotochigova *et al.*, Phys. Rev. A 55, 191 (1997)

Comparison with precise measurements





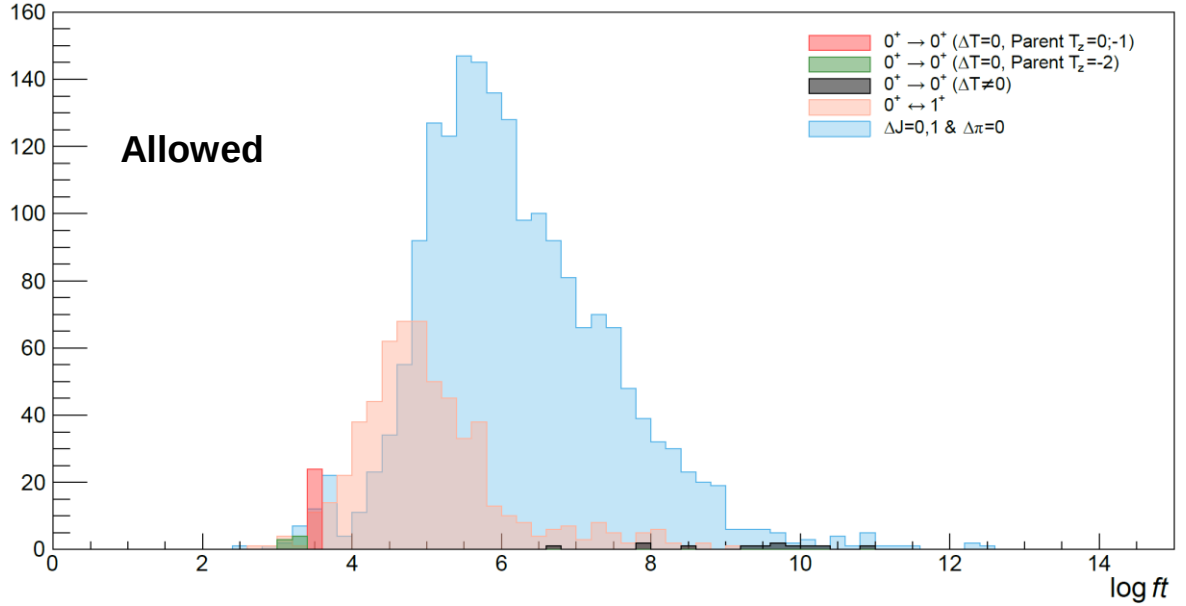
New review of log-*ft* values

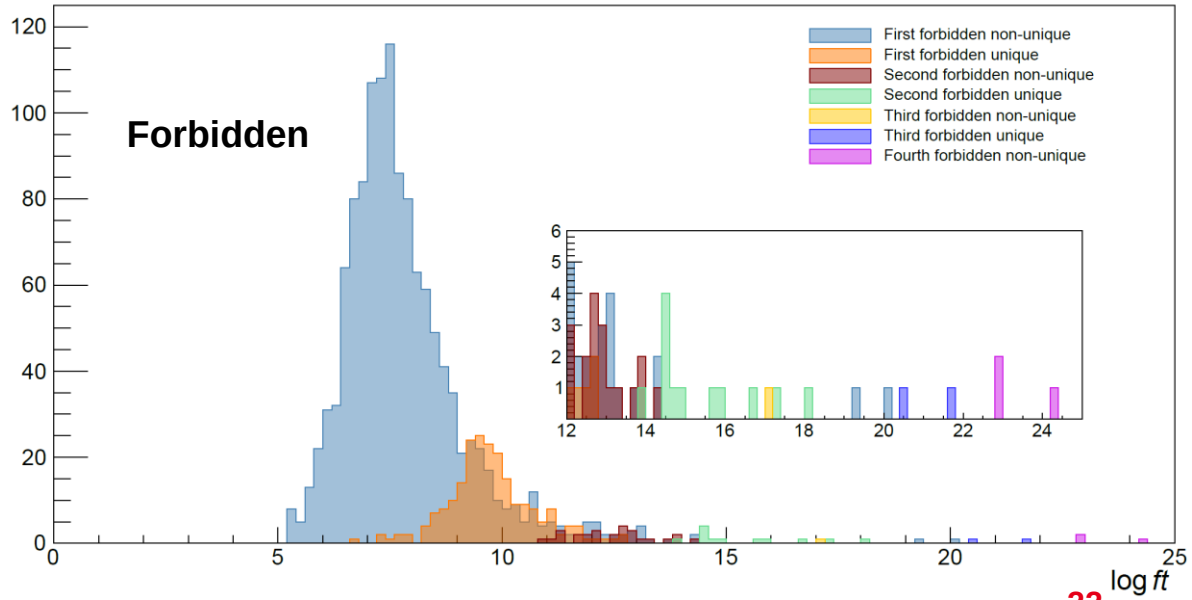
Former review of log-*ft* values, calculated with the LogFT code: *B. Singh et al., Nuclear Data Sheets 84, 487 (1998)*

These values are used in nuclear structure studies, e.g. to assign spin and parity to a level.

Collaborative work: B. Singh (McMaster University), S. Turkat and K. Zuber (TU Dresden), X. Mougeot (CEA-LNHB)

- ✓ Update of B and EC decays present in ENSDF database (as of mid-April 2023).
- ✓ Use of BetaShape to calculate the log-*ft* values.
- ✓ In total, 26 318 transitions calculated. Selection of well-defined transitions. Possible pandemonium nuclei flagged.
- ✓ 4 038 transitions survived this filtering. All distributions re-established. Specific transitions are discussed.
- ✓ *S. Turkat et al., Atomic Data and Nuclear Data Tables 152, 101584 (2023)*





Evaluated nuclear decay data

Argonne NATIONAL LABORATORY

Member of the US Nuclear Data Program

Adoption of the BETASHAPE code

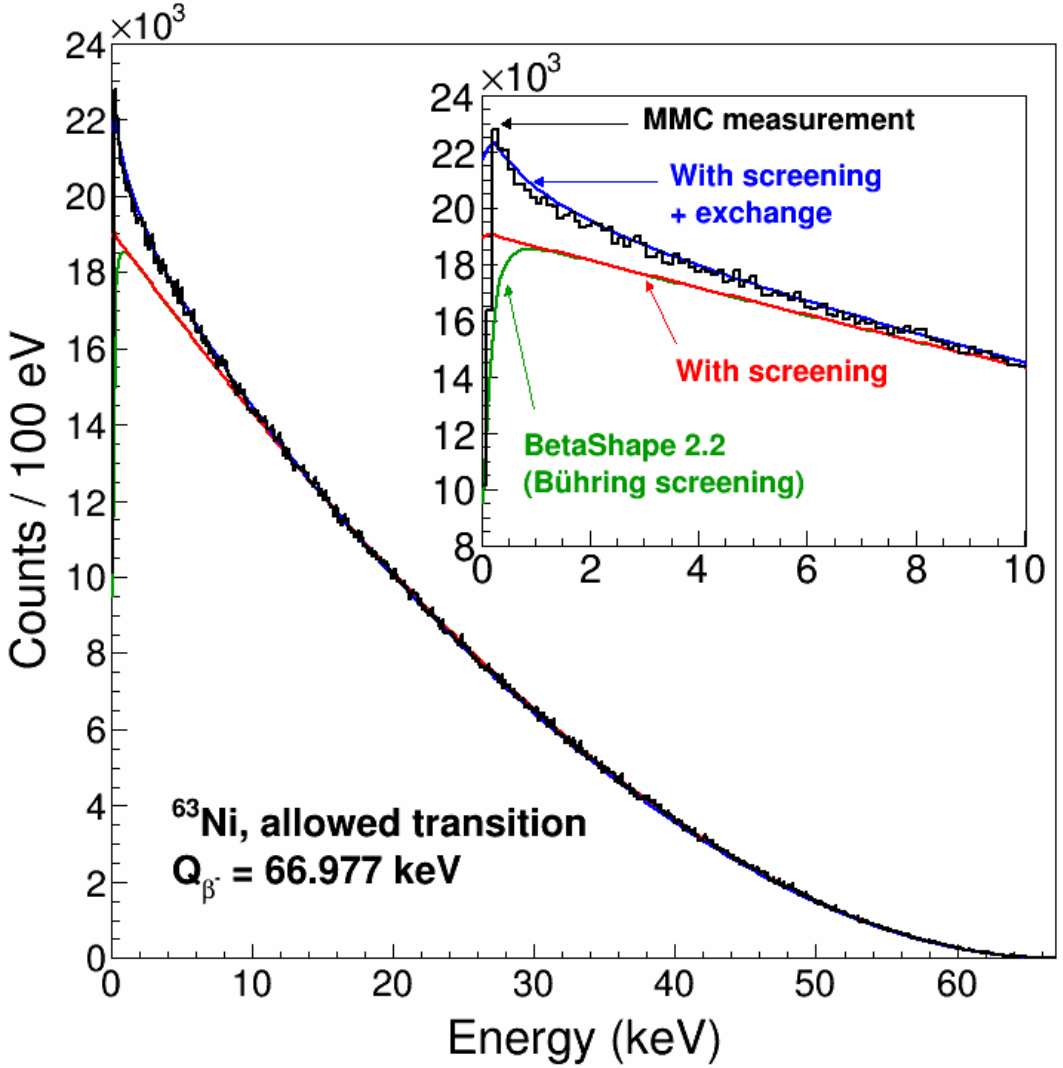
T. Kibédi (ANU), F.G. Kondev (ANL), A. Nichols (U Surrey),
B. Singh (McMaster U) & X. Mougeot (CEA-LNHB)

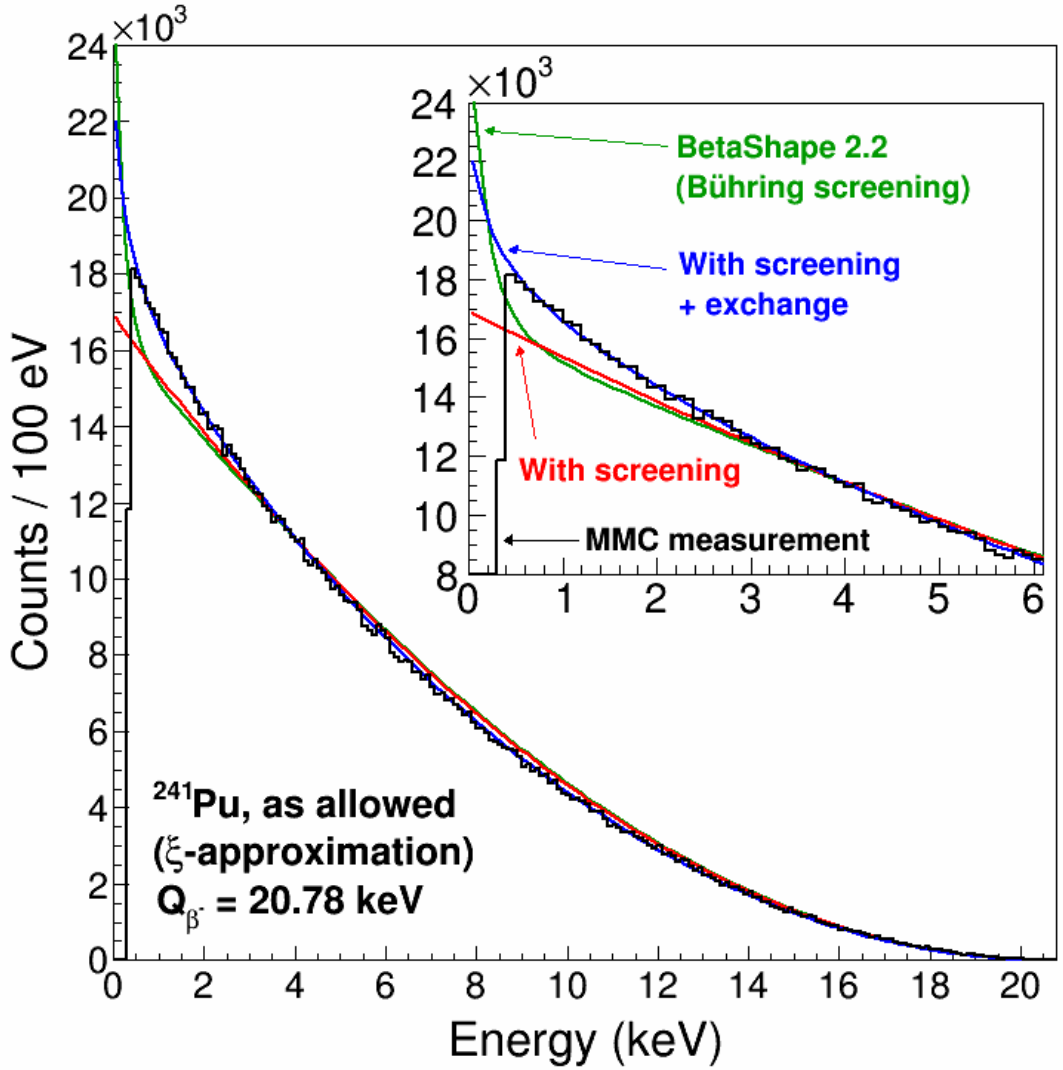
NSDD Meeting, October 24-28, 2022, Canberra, Australia

U.S. DEPARTMENT OF ENERGY

- ✓ The BetaShape code was already adopted several years ago by the DDEP collaboration for their decay data evaluations.
- ✓ The version 2.3 of BetaShape was adopted by the NSDD network for all ENSDF evaluations.
- ✓ Specific requests are regurlarly formulated by the NSDD evaluators to match ENSDF policies. Versions 2.3 and 2.4 (current) include all these requests. Examples: accurate atomic effect; fast calculations (via tabulations); non-numerical uncertainties; asymmetric uncertainties.

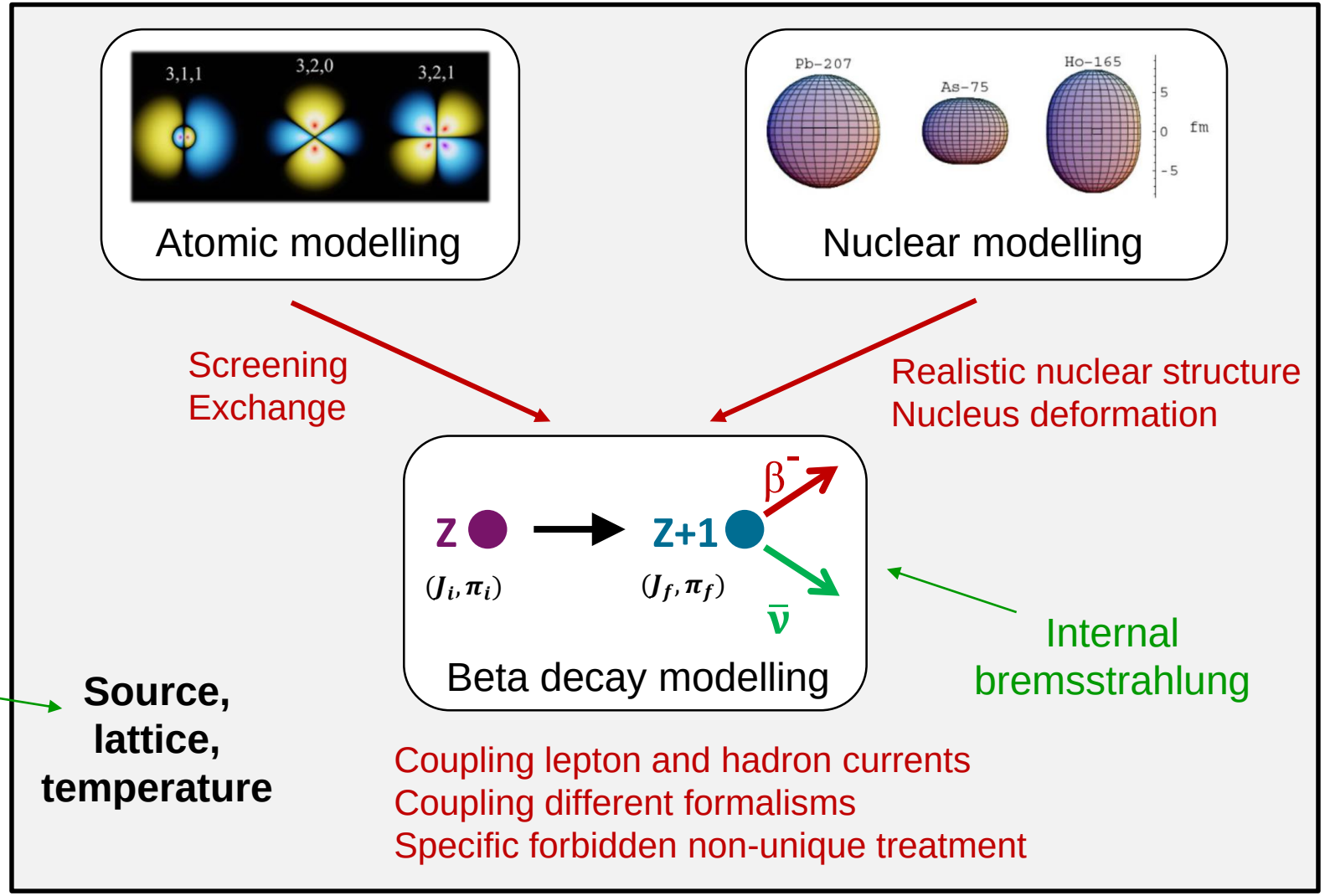
Atomic screening and exchange effects





Conclusion

Overview


3,1,1
3,2,0
3,2,1
Atomic modelling
Pb-207
As-75
Ho-165
5
0
-5
fm
Nuclear modelling
Screening
Exchange
Realistic nuclear structure
Nucleus deformation
Z
Z+1
β^-
$\bar{\nu}$
(J_i, π_i)
(J_f, π_f)
Beta decay modelling
Additional effects
Environmental influence
Source, lattice, temperature
Internal bremsstrahlung
Coupling lepton and hadron currents
Coupling different formalisms
Specific forbidden non-unique treatment

Take away

Beta decays are of importance for nuclear decay data, but also for a wide range of scientific topics from fundamental physics to applications.

- **Allowed and forbidden unique transitions can be calculated** without any nuclear structure **with good accuracy**. Except in a few specific cases (“accidental cancellation of nuclear matrix elements”).
- If targeted accuracy better than 1-5%, detailed models are necessary.
- ✓ When no nuclear structure is required, the BetaShape code already improves the situation.

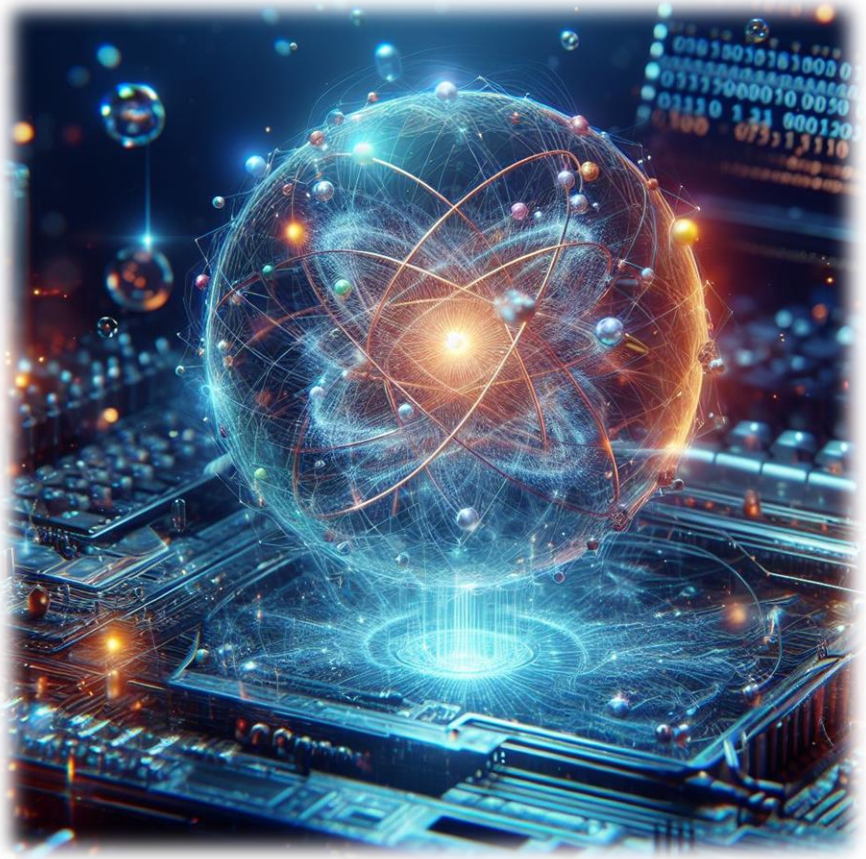
  → Available on IAEA-NSDD GitHub: https://github.com/IAEA-NSDDNetwork

- **Forbidden non-unique transitions are sensitive to nuclear structure**. Some can be calculated accurately as allowed or forbidden unique.

Ongoing developments and challenges

- **High-precision measurements**, which can be used as a probe of nuclear models.
- **Medium and heavy nuclei, nuclear deformation**

  → Ongoing collaboration in the framework of the European metrology project PrimA-LTD.
- **Effective values of weak interaction coupling constants** is still an open subject.



Thank you for your attention

cea

université PARIS-SACLAY

LNE LABORATOIRE NATIONAL DE MÉTROLOGIE ET D'ESSAIS

LNHB Laboratoire National Henri Becquerel

EURAMET